Assortment Planning under Uncertainty in Physical Internet Enabled Supply Chains

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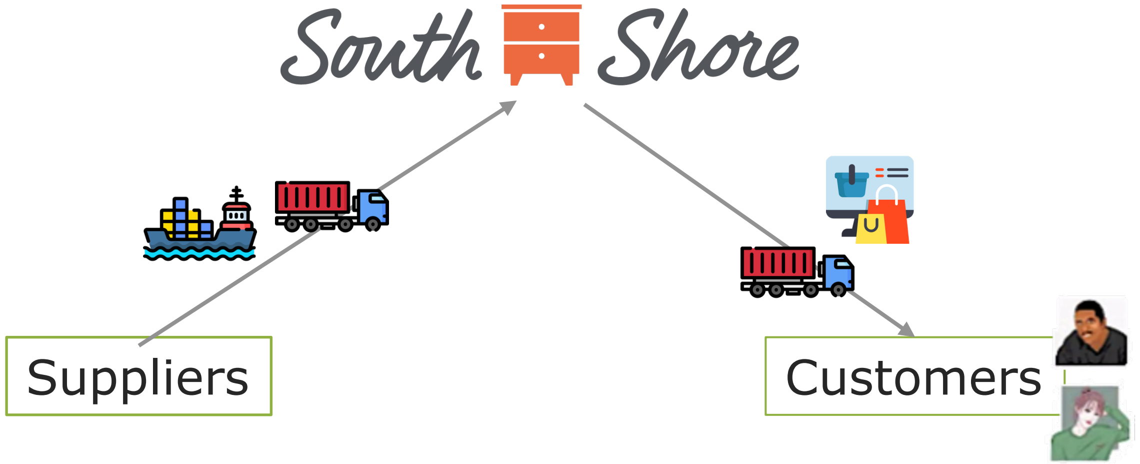
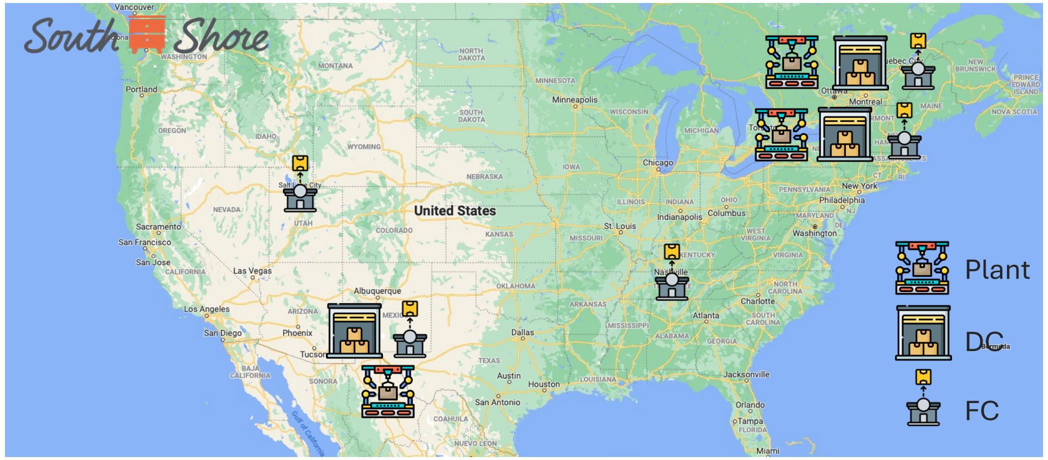
IPIC 2024 Ph.D. Colloquium

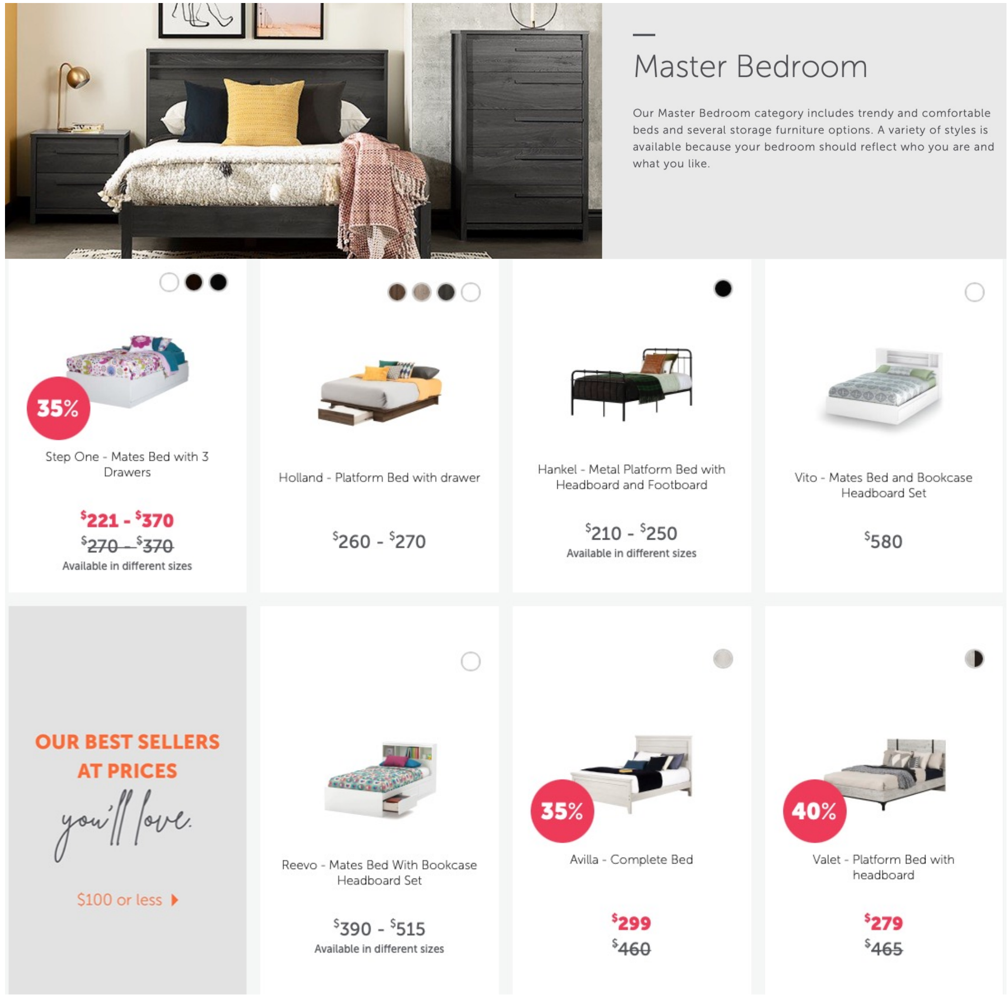
Georgia Tech Physical Internet Center

Introduction

Motivation: Overview of the Industry Case







Motivation: Pandemic-induced Disruptions

-35%

In the number products offered

The pandemic-induced disruptions in production and labor shortages necessitated a significant reduction the product assortment size.

Active in-stock products decreased by about 35% (from 1450 to 950) from March to June 2020.

We aim to explore decision-making strategies for product assortment under production capacity limitations and to reassess these decisions in response to changes or disruption.

Assortment Planning Problem

:Decision on which subset of items to offer to customers to maximize the expected profit

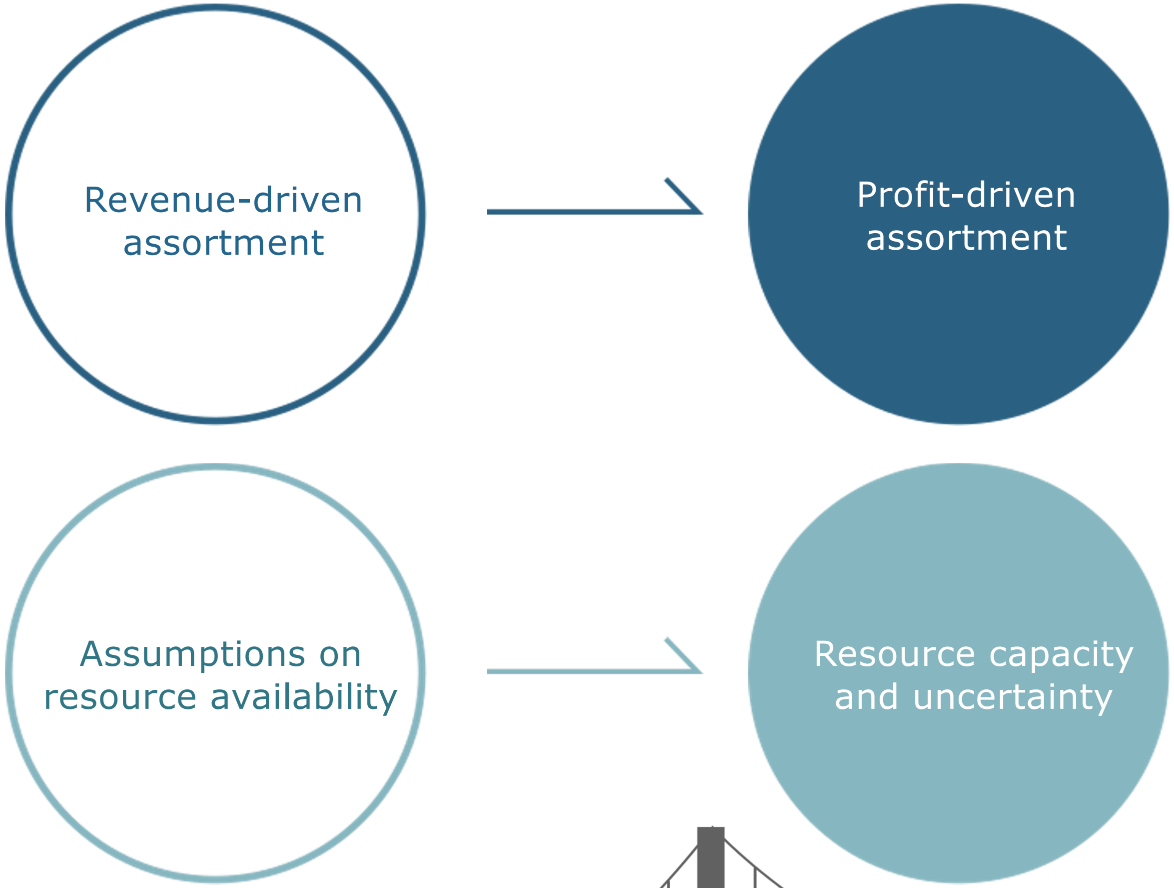
Challenges with the trade-off in increasing variety and operational costs.

The constant widening of product selection has been resulting in inefficient product assortment.

Over 70% of all sales coming from 2% of SKUs (Giménez 2021)

Bridging the Gap Between Marketing and Operations

Assortment decisions are often made by the marketing department, while intricately linked to operational decisions

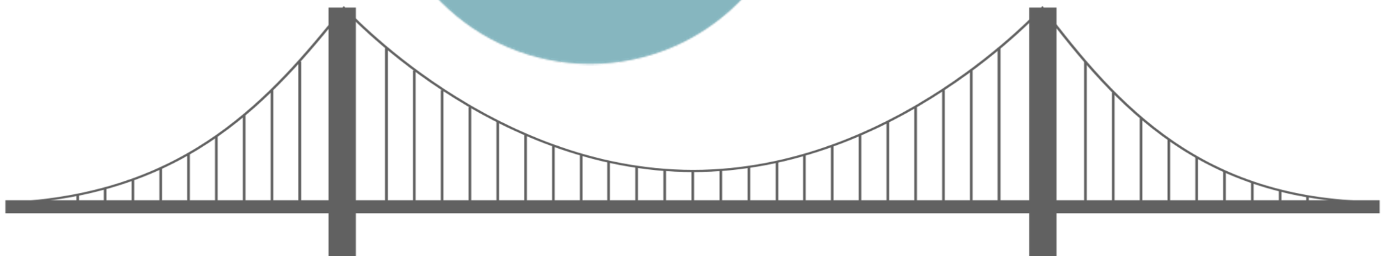


Assortment decisions made with supply chain costs

- ✓ Tradeoff between revenue and operational costs
- ✓ 71% of global companies highlight raw material costs as their number one supply chain threat (KPMG 2023)

Assortment decisions made with supply chain capacities

- ✓ Increasing resource limitations
- ✓ Joint planning can deliver up to 20% in incremental sales and profits (Giménez 2021)



Assortment Planning under Uncertainty

Supplier-driven

Interrupted raw materials supplies in and suspensions in manufacturing operations

From geopolitical conflicts, climate change weather events, pandemics, etc.

Volatility in key commodity prices and availabilities (KPMG 2023)

Upstream Uncertainty

Downstream Uncertainty

Customer-driven

Fast paced changes in market trends and volatile customer demand

Assortment Planning with Customer-driven Substitution

Customers are frequently willing to buy a similar product if their first-choice option is not available (Shin et al. 2015)

Substitution refers to the use of one product to satisfy demand for a different product

01

Reduced Assortment

Not all products are available from capacity constraints or for profitability

02

Product Substitution

Customers may choose to substitute, inducing risk-pooling effects (Chopra and Meindl 2010, Shin et al. 2015)

03

Assortment and Substitution

Scarcity of models with substitution when capacitated or with supply chain considerations

04

We leverage the **customer product substitution** behavior from the procurement and manufacturing stage to decide the most efficient, **profit-maximizing assortment.**

Physical Internet Center

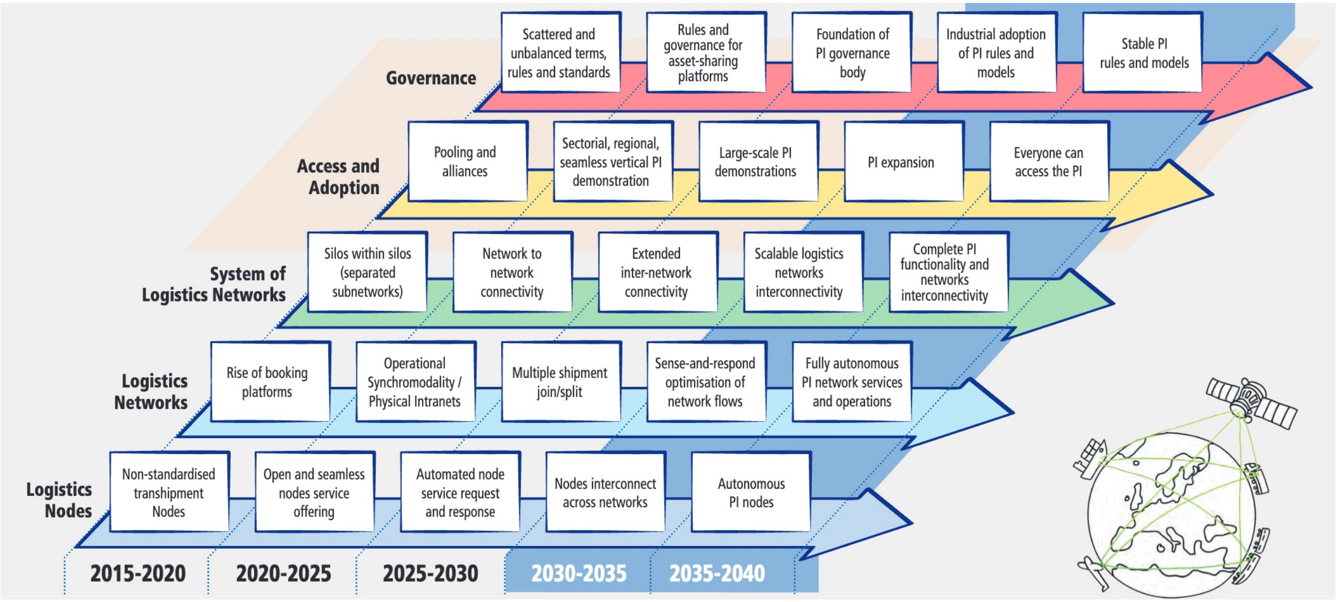
Replace numbers with icons?

Supply Chain and Logistics Institute

Physical Internet Enabled Supply Chains

Universal interconnectivity (interconnectivity in multiple layers including physical, digital, operational, transactional, and legal interconnectivity) is key to making the Physical Internet an open, global, efficient and sustainable system (Montreuil et al. 2012)

Physical Internet Enabled Supply Chains

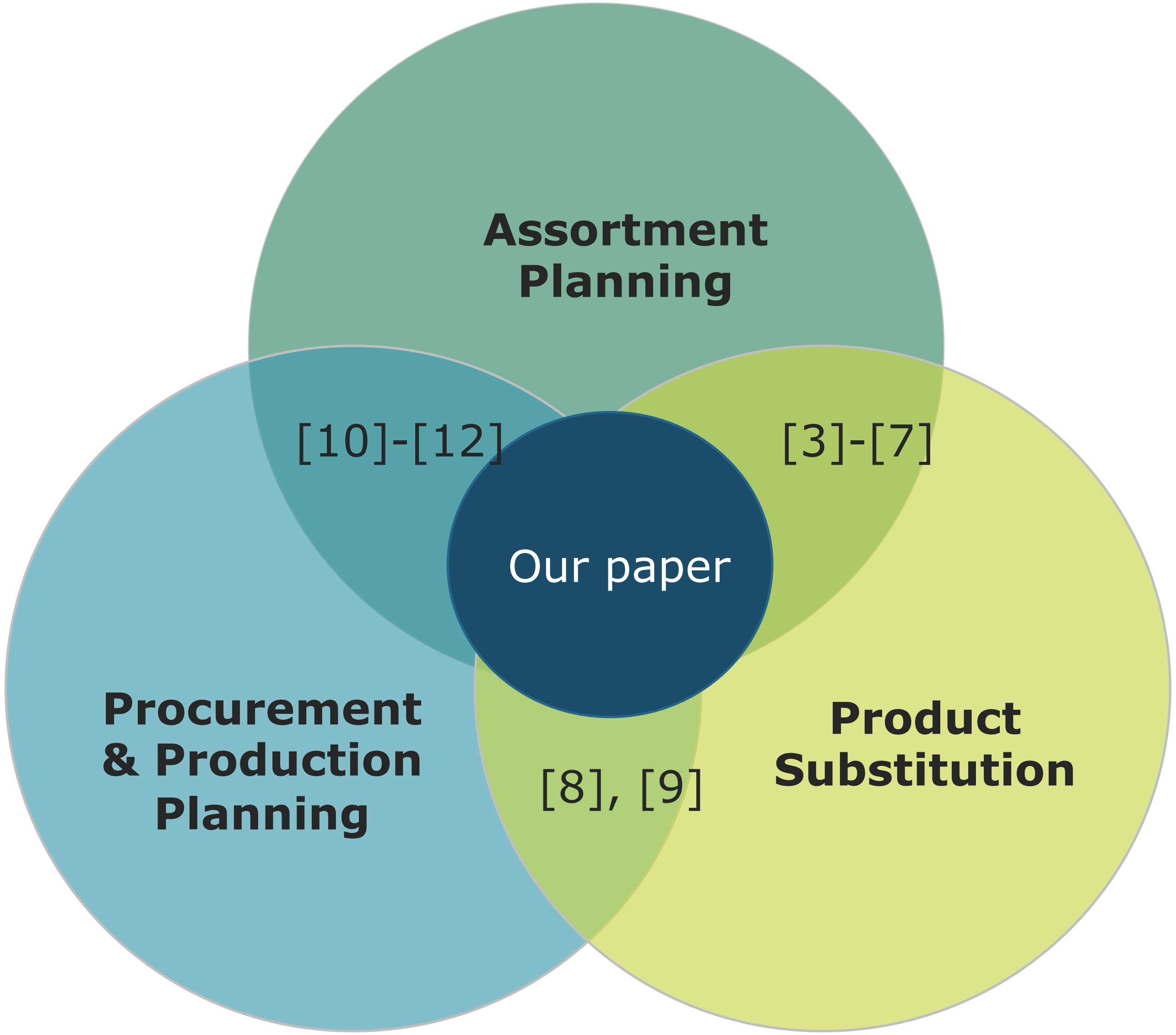


Moving through different phases of the Physical Internet thus provides a greater flexibility in delivery options, allowing for a full exploitation of the conversion curves

With faster delivery, the left-side boundary of the conversion curve shifts to further left, and with effective deployment, delivery can be delayed and even finetuned as needed

Position in Literature

#	Authors
1	Yuc̈el et al.
2	Zeppetella et al.
3	Çömez-Dolgan et al.
4	Kök and Fisher
5	Transchel et al.
6	Hübner et al.
7	Smith and Agrawal
8	Sereshti et al.
9	Rao et al.
10	Dobson and Yano
11	Morgan et al.
12	Andrede et al.



Problem Description

Problem Definition

> We take an integrative approach for an assortment planning problem for a manufacturer-retailer in a make-to-stock environment

Multiple Review Periods

The set of products offered do not change abruptly and frequently

Assortment decisions made for a longer time horizon

Industries with durable goods

Customer-driven Product Substitution

Customer substitution for risk-pooling effect and profit maximization

Leveraged from procurement and manufacturing stages

Exogenous substitution model

Supply Chain Considerations

Assortment costs

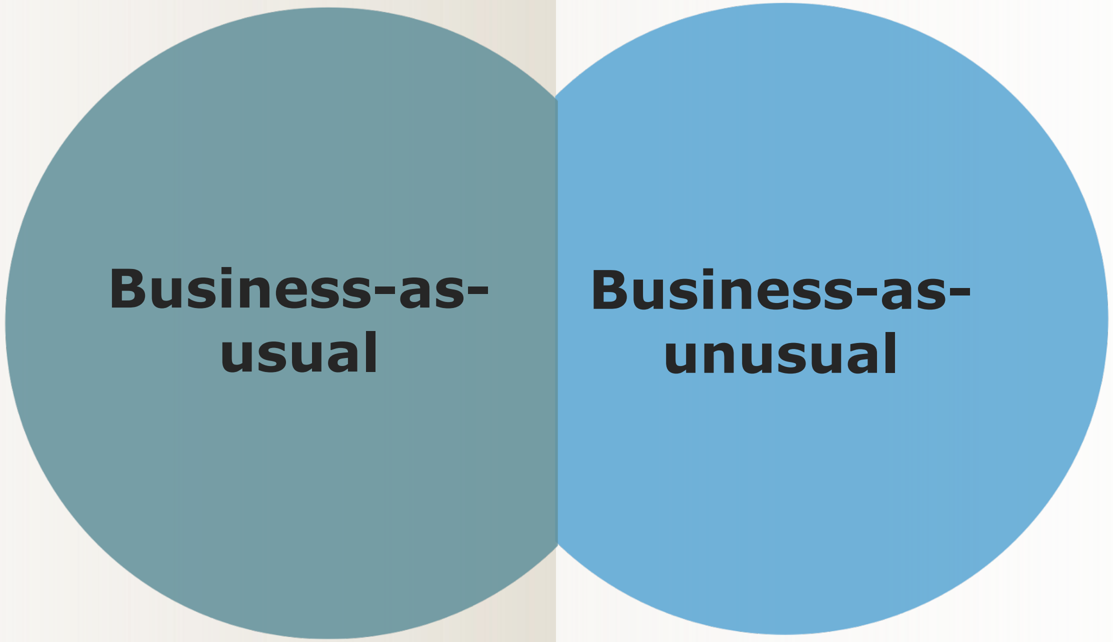
Procurement, production, and inventory costs

Procurement, production, and inventory capacities

Problem Definition

We aim to maximize the expected profit through optimizing supply and demand alignment considering the following scenarios:

We propose a profit-maximizing assortment with *supply chain considerations* and customer-driven product substitution.



We ensure customer service in disruptive retailing environments with *dynamic, adaptive assortments* through customer-driven product substitution.

Problem Definition

Goal:

Select the product assortment from all possible products $\mathrm{i} \in I$ for time periods $t \in \{1, \dots, T\}$ to maximize the expected total profit as the difference between expected revenue and associated costs given supply and demand uncertainty

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$$\boldsymbol{TR} - (\mathbf{TCO} + \boldsymbol{TCS} + \boldsymbol{TCP} + \boldsymbol{TC}\mathbf{I})$$

Problem Definition

Do we keep the scenario notations?
Or not after the next slide?

Goal:

Select the product assortment from all possible products $i \in I$ for time periods $t \in \{1, \dots, T\}$ to maximize the expected total profit as the difference between **expected revenue** and **associated costs** given supply and demand uncertainty

$$TR(\omega) - (\mathrm{TCO}(\omega) + TCS(\omega) + TC\mathrm{P}(\omega) + TC\mathrm{I}(\omega))$$

Problem Definition

Goal:

Select the product assortment from all possible products $i \in I$ for time periods $t \in \{1, \dots, T\}$ to maximize the expected total profit as the difference between **expected revenue** and **associated costs** given supply and demand uncertainty

$$\boldsymbol{TR}(\boldsymbol{\omega}) - (\mathbf{TCO}(\boldsymbol{\omega}) + \boldsymbol{TCS}(\boldsymbol{\omega}) + \boldsymbol{TCP}(\boldsymbol{\omega}) + \boldsymbol{TC}\mathbf{I}(\boldsymbol{\omega}))$$

Decisions:

Assortment: $Z_{it} \in \{0,1\}$, $Z_{it}^{+} \in \{0,1\}$, $Z_{it}^{-} \in \{0,1\}$

Procurement planning: $O_t^s \in \{0,1\}$, $X_{jt}^s \geq 0$

Production planning: $X_{it} \geq 0$

Inventory planning: I_{it}, $I_{jt} \geq 0$

Demand fulfillment: $d0_{it}$, ds_{ikt}, $dl_{it} \geq 0$

Constraints:

Supplier capacities: $KS_{jt}^s(\omega)$

Production capacities: KS_{jt}^s

Product recipes: u_{ji}

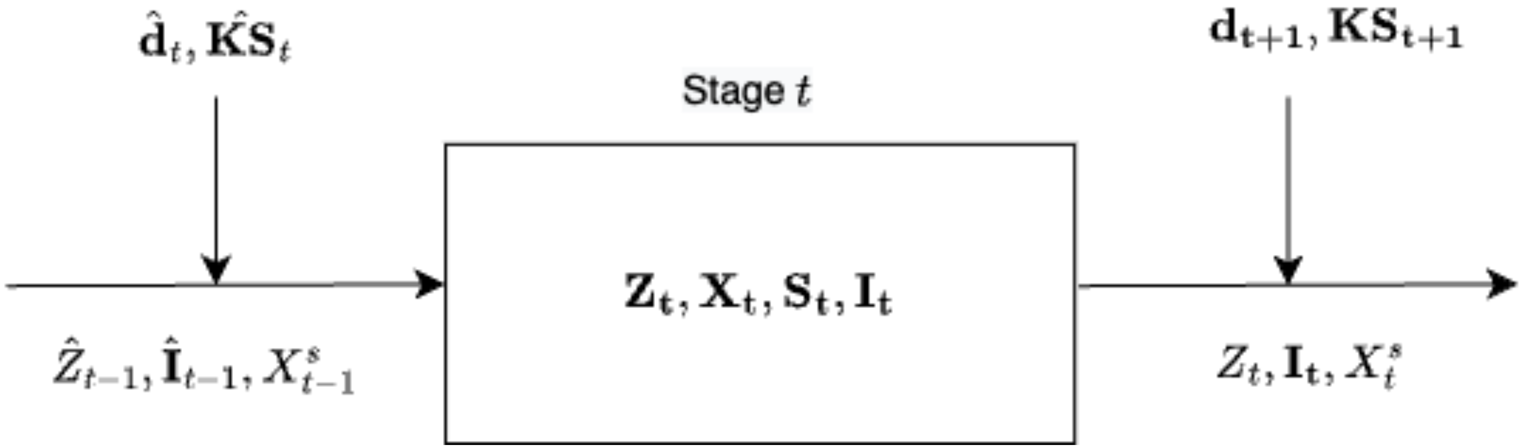
Inventory capacities: KI_t, KI_t^s

Customer behavior & substitution: γ_{ik}

Flow balance constraints

Economies of scale/non-linear cost functions

Dynamic Supply Chain Driven Assortment Planning

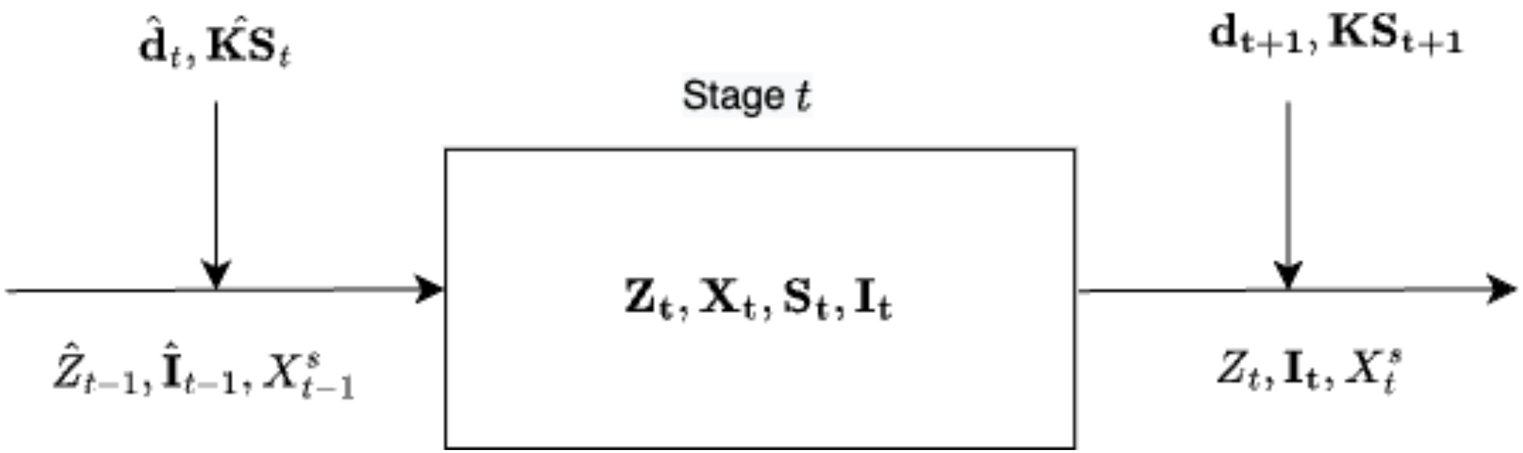


where state is given as $S_t = (Z_{t-1}, \boldsymbol{I}_{t-1}, X^S_{t-1})$ and action as $a_t = (\boldsymbol{Z}_t, \boldsymbol{X}_t)$ for each period t.

$$\max_{a_t \in \mathcal{A}_t} \left(\mathbb{E}_{\omega \in \Omega} \sum_{t \in \mathcal{T}} F_t(S_t, a_t, \boldsymbol{d}_t(\omega), \boldsymbol{B}_t(\omega) | S_0) \right)$$

Curse of dimensionality → quickly becomes intractable

Dynamic Supply Chain Driven Assortment Planning



where state is given as $S_t = (Z_{t-1}, \boldsymbol{I}_{t-1}, X^S_{t-1})$ and action as $a_t = (\boldsymbol{Z}_t, \boldsymbol{X}_t)$ for each period t.

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Curse of dimensionality → quickly becomes intractable

Our Contributions

Problem

Supply-chain-driven assortment planning under uncertainty

Model

Multi-stage stochastic programming model

Substitution

MNL-based closed-form expression for substitution probabilities

Managerial Insights

Value of integrative approach and substitution

Our Contributions

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Managerial Insights

Value of integrative approach and substitution

We are the first to address the supply-chain-driven assortment planning problem under uncertainty with:

i. Stochastic, non-stationary demand
ii. Uncertainty in supply capacities
iii. Customer-driven substitution leveraged from procurement and manufacturing stages
iv. Multi-period assortment decisions
v. Procurement, production, and inventory considerations

Our Contributions

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Managerial Insights

Value of integrative approach and substitution

We formulate the problem as a multi-stage stochastic model for assortment decisions over the planning horizon and propose a two-stage stochastic program approximations based on a rolling horizon approach.

Our approach allows for scalability and practical application in real-life industry settings, an aspect that remains largely unexplored in literature (Hübner et al. 2016)

Our Contributions

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MNL-based closed-form expression for substitution probabilities

Managerial Insights

Value of integrative approach and substitution

We propose an MNL-based closed form expression for estimating exogenous product substitution probabilities:

> Based on sales and availability data

> Parsimonious estimation; computational advantage over existing methods for estimating substitution probabilities

Our Contributions

Problem

Supply-chain-driven assortment planning under uncertainty

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Multi-stage stochastic programming model

Substitution

MNL-based closed-form expression for substitution probabilities

Managerial Insights

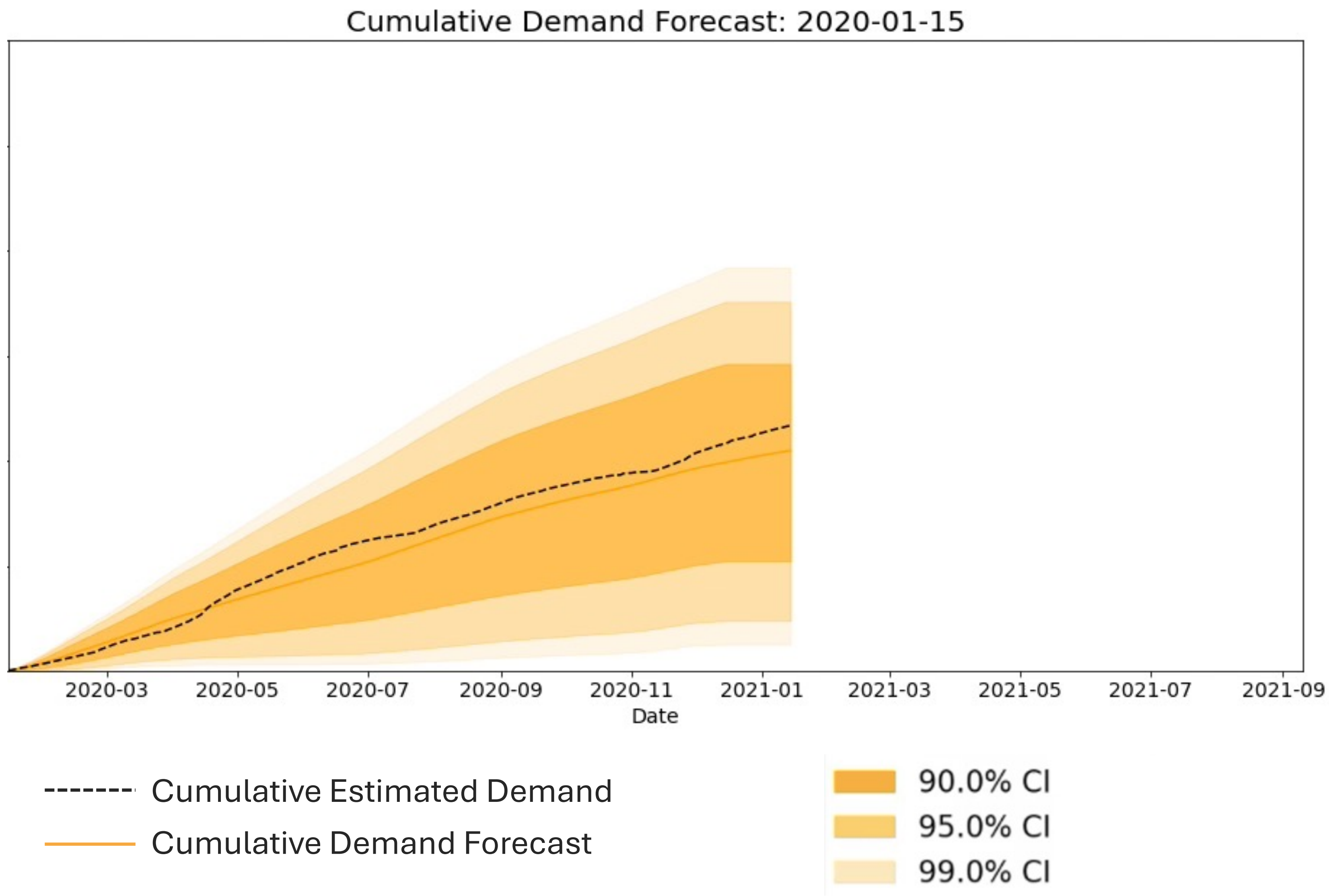
Value of integrative approach and substitution

Inspired by a real industrial case, we provide important managerial insights, specifically on:

> Value of taking an integrative approach on assortment planning

> Value of leveraging customer-driven product substitution

Customer Demand as a Multi-period Lookahead Forecast



Demand is oftentimes volatile and time-varying > Stochastic, non-stationary demand

Demand forecast is updated as sales information is updated

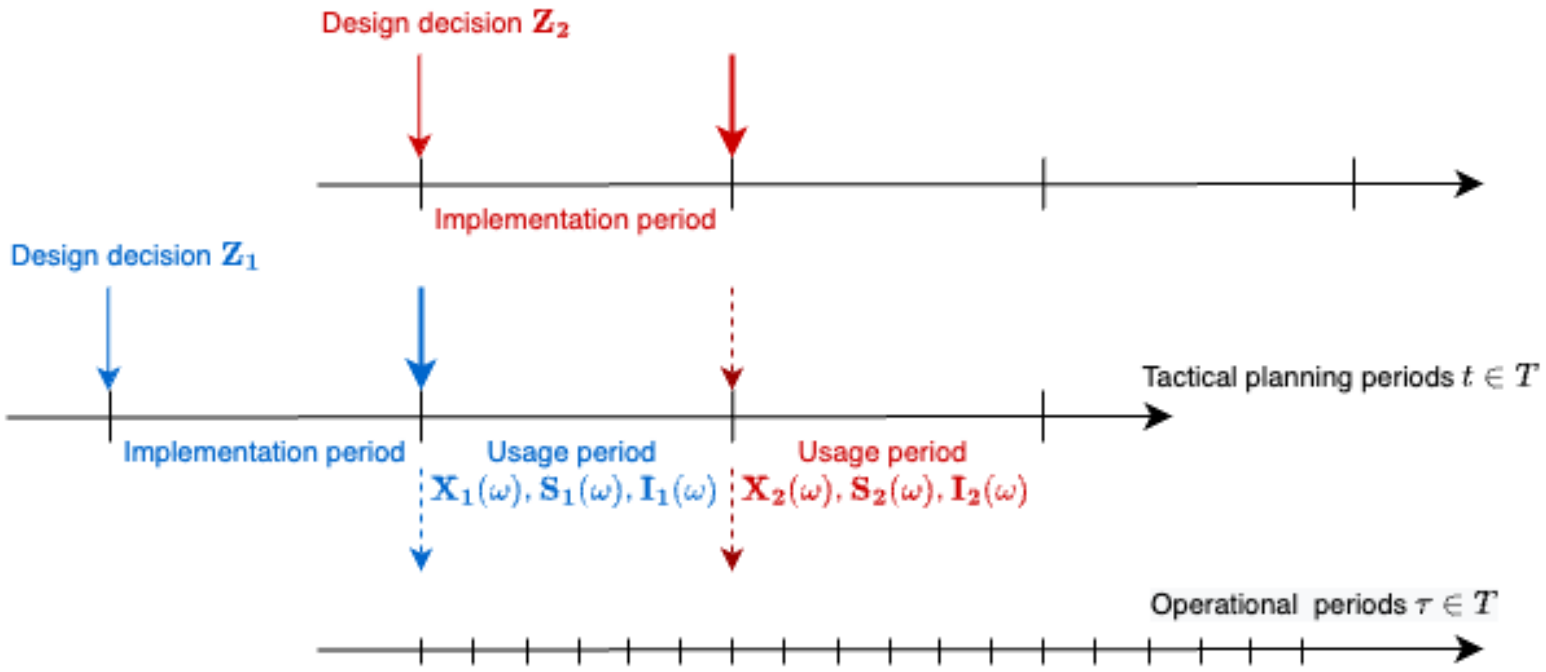
Further necessitates the review of the assortment decisions as the lookahead forecast updated each period

Model Formulation

Multi-stage Stochastic Programming Model

> Assortment decisions are *tactical*

> Information asymmetry requires an *anticipation of the future* and the values of the operational decisions



$$\max \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} \phi_\omega \left(TR_t(\omega) - TCO_t(\omega) - TCA_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^s(\omega) - TCI_t^p(\omega) \right)$$

ϕ_ω denotes the probability of scenario $\omega \in \Omega_t$
Ω_t denotes the set of distinct restricted scenarios up to stage t

Multi-stage Stochastic Programming Model

$$\max \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} \phi_\omega \left(TR_t(\omega) - TCO_t(\omega) - TCA_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^s(\omega) - TCI_t^p(\omega) \right)$$

s.t.

$$TR_t(\omega) = \sum_{i \in \mathcal{I}} r_i \left(X_{it}(\omega) + I_{it}(a(\omega)) - I_{it}(\omega) \right) \forall t \in \mathcal{T}, \omega \in \Omega_t$$

Total Revenue

$$TCA_t(\omega) = \left(ac_i^+ Z_{it}^+(\omega) + ac_i^- Z_{it}^-(\omega) + pc_i Z_{it}(\omega) \right) \forall t \in \mathcal{T}, \omega \in \Omega_t$$

Assortment Costs

$$TCO_t(\omega) = \sum_{s \in \mathcal{S}} oc_s O_{st}(\omega) \forall t \in \mathcal{T}, \omega \in \Omega_t$$

$$TCS_t(\omega) = \sum_{j \in \mathcal{J}} \sum_{l=1}^{m_{sj}} \left(y_j^l Y_{jt}^{sl}(\omega) + c_j^{sl} \left(X_{jt}^{sl}(\omega) - \bar{X}_j^{sl-1} Y_{jt}^{sl}(\omega) \right) \right) \forall t \in \mathcal{T}, \omega \in \Omega_t$$

Procurement Costs

$$TCP_t(\omega) = \sum_{i \in \mathcal{I}} \sum_{l=1}^{m_i} \left(y_i^l Y_{it}^l(\omega) + c_i^l \left(X_{it}^l(\omega) - \bar{X}_i^{l-1} Y_{it}^l(\omega) \right) \right) \forall t \in \mathcal{T}, \omega \in \Omega_t$$

Production Costs

$$TCI_t^s(\omega) = \sum_{j \in \mathcal{J}} \frac{\sum_{s \in \mathcal{S}} X_{jt}^s(\omega) + I_{jt}(a(\omega)) - I_{jt}(\omega)}{2} h_j \forall t \in \mathcal{T}, \omega \in \Omega_t$$

$$TCI_t^p(\omega) = \sum_{i \in \mathcal{I}} \frac{\sum_{s \in \mathcal{S}} X_{it}(\omega) + I_{it}(a(\omega)) - I_{it}(\omega)}{2} h_j \forall t \in \mathcal{T}, \omega \in \Omega_t$$

Inventory Costs

Multi-stage Stochastic Programming Model

$$\max \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega_t} \phi_\omega\left(TR_t(\omega) - TCO_t(\omega) - TCA_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^s(\omega) - TCI_t^p(\omega)\right)$$

s.t.

$$PAR_t(Z_t(\omega)) \leq \alpha \; \forall t \in \mathcal{T}$$

Business Requirement on Availability

$$Z_{it}(\omega) - Z_{it}(a(\omega)) \leq Z_{it}^{+}(\omega) \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega_t$$
$$Z_{it}(a(\omega)) - Z_{it}(\omega) \leq Z_{it}^{-}(\omega) \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega_t$$

Assortment Constraints

Multi-stage Stochastic Programming Model

$$\max \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} \phi_\omega \left(TR_t(\omega) - TCO_t(\omega) - TCA_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^s(\omega) - TCI_t^p(\omega) \right)$$

s.t.

Procurement capacity met with a lead time

Approximating procurement costs to account for economies of scale

Procurement Constraints

Production capacity for active products

Approximating production costs to account for economies of scale

Production Constraints

Multi-stage Stochastic Programming Model

$$\max \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} \phi_\omega \big(TR_t(\omega) - TCO_t(\omega) - TCA_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^s(\omega) - TCI_t^p(\omega)\big)$$

s.t.

Demand met through primary/substitute products
Or results in lost sales

Product substitution

Inventory capacity

Finished products flow balance

Raw material flow balance

Sales and Substitution

Inventory Constraints

Multi-stage Stochastic Programming Model

$$\max \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} \phi_\omega \big(TR_t(\omega) - TCO_t(\omega) - TCA_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^s(\omega) - TCI_t^p(\omega) \big)$$

s.t.

Demand met through primary/substitute products
Or results in lost sales

Product substitution

Sales and Substitution

Organize by products/rm?

Inventory capacity

Finished products flow balance

Raw material flow balance

Inventory Constraints

Multi-stage Stochastic Programming Model

$$\max \sum_{t \in \mathcal{T}} \sum_{\omega \in \Omega_t} \phi_\omega \left(TR_t(\omega) - TCO_t(\omega) - TCA_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^s(\omega) - TCI_t^p(\omega) \right)$$

s.t.

$$Z_{it}(\omega), Z_{it}^{+}(\omega), Z_{it}^{-}(\omega) \in \{0,1\} \;\; \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega_t$$
$$O_{st}(\omega) \in \{0,1\} \;\; \forall s \in \mathcal{S}, t \in \mathcal{T}, \omega \in \Omega_t$$
$$Y_{jt}^{sl}(\omega) \in \{0,1\} \;\; \forall s \in \mathcal{S}, j \in \mathcal{J}, t \in \mathcal{T}, l = 1, \dots, m_{sj}, \omega \in \Omega_t$$
$$Y_{it}^{l}(\omega) \in \{0,1\} \;\; \forall i \in \mathcal{I}, t \in \mathcal{T}, l = 1, \dots, m_i, \omega \in \Omega_t$$
$$X_{jt}^{s}(\omega), I_{jt}(\omega) \geq 0 \;\; \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega_t$$
$$X_{it}(\omega), I_{it}(\omega), S_{it}^{0}(\omega), S_{it}^{l}(\omega) \geq 0 \;\; \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega_t$$
$$S_{kit}^{s}(\omega) \geq 0 \;\; \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega_t$$

Domain of decision variables

Two-stage Stochastic Programming Approximation

The design decisions for all periods are transferred to the first stage

Assortment related Z_1 variables for the first stage represent the here-and-now decisions

$$\max_{Z,X,S,I} \mathbb{E}_{\omega\in\Omega} h(\boldsymbol{Z},\ \omega) = \max \sum_{t\in\mathcal{T}} \left(\mathbb{E}_{\Omega_t} Q_t(Z,\omega) - TCA_t\right)$$

s.t.

$$TCA_t = \sum_{i\in\mathcal{I}} (ac_i^{+} Z_{it}^{+} + ac_i^{-} Z_{it}^{-} + pc_i Z_{it})\ \forall t\in\mathcal{T}$$

$$PAR_t(Z_t) \leq \alpha\ \forall t\in\mathcal{T}$$

$$Z_{it} - Z_{i,t-1} \leq Z_{it}^{+} \forall i\in\mathcal{I}, t\in\mathcal{T}$$

$$Z_{i,t-1} - Z_{it} \leq Z_{it}^{-} \forall i\in\mathcal{I}, t\in\mathcal{T}$$

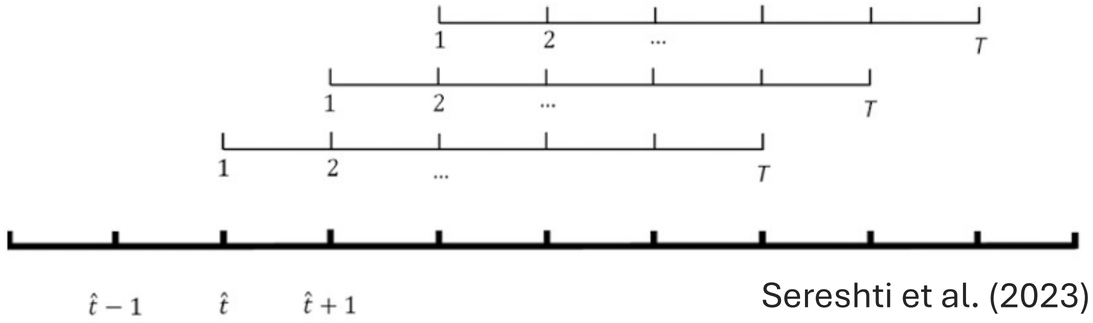
$$Z_{it}\ \ Z_{it}^{+}, Z_{it}^{-} \in \{0,1\} \forall i\in\mathcal{I}, t\in\mathcal{T}$$

Assortment related first-stage constraints

Where $Q_t(Z,\omega)$ is the solution of the second-stage problem:

$$Q_t(Z,\omega) = \max_{\boldsymbol{X},\boldsymbol{S},\boldsymbol{I}} \left(TR_t(\omega) - TCO_t(\omega) - TCS_t(\omega) - TCP_t(\omega) - TCI_t^{s}(\omega) - TCI_t^{p}(\omega)\right)$$

Two-stage Model and Rolling Horizon Framework



Sereshti et al. (2023)

Rolling-horizon framework

> Decisions optimized in period 0 by considering the first H periods

> Only the decisions of stage 0 are implemented

> Demand and supplier information is revealed for period 0

> Re-optimize assortment decisions for horizon 1 to $H + 1$

> Process is repeated until the last period

Preliminary Results

Conclusion

Conclusion and Next Steps

- Point 1
- Point 2
- Point 3

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References and Acknowledgments