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Towards the Physical Internet with Coloured Petri Nets

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Abstract: *Coloured Petri Nets can be a valuable and powerful tool to design, analyse, and control the subsystems composing the Physical Internet, as they are able to capture the precedence relations and interactions among events which characterize the facilities and infrastructures (multimodal logistics centres and hubs, transit centres, roads and railways) through which π-containers are delivered. In this paper, the use of Coloured Petri Nets in the field of the Physical Internet is discussed and an example of the application of such a modelling tool to a multimodal hub in the PI is provided. The multimodal hub consists of four areas: a port area at which vessels arrive and depart, a train terminal for rail transportation, a road terminal for truck-to-X (and vice-versa) transhipment, and a storage area. The storage area and the road terminal are considered in detail, and two nets representing a section of a π-conveyor and a π-sorter/π-composer are proposed to illustrate the applicability of the CPN formalism to the Physical Internet paradigm.*

Keywords: *multimodal hubs; π-containers management; modelling tools; coloured Petri nets; simulation tools*

1 Introduction

In the next decades, the road to the Physical Internet (PI) will change drastically most of the production and logistic processes that actually characterize the supply chain. New multimodal logistic centres able to handle modular containers, open networks in which the various actors of the supply chain share transport services, delivery of disaggregated goods through alternate paths in analogy with data packets transmitted through the digital internet, are some of the newly features provided by the PI (Montreuil, 2011). In this framework, technicians and scientists are working together to provide enabling technologies and powerful ICT services, and a big challenge of researches is to provide effective modelling tools which allow analysing the performance of such kind of interoperating systems.

Most of the subsystems composing the PI are actually discrete-event systems (DESs) as they are characterized by the presence of concurrent and asynchronous events which influence their states (Cassandras and Lafortune, 2010). Among the modelling tools for DESs, Petri nets (PNs) have been proven to be a valuable and powerful tool for design, analysis, and control of DESs (Murata, 1989). PNs have been defined in the 1960s by Carl Adam Petri (Petri, 1962) and since then hundreds of researchers have adopted such a modelling tool to represent, analyse and control specific classes of DESs such as manufacturing and production systems (Desrochers, 1990; DiCesare et al., 1993, Zhou and DiCesare, 1993), communication protocols (Suzuki et al. 1990; Berthomieu and Diaz, 1991; Billington and Han, 2007), indoor transportation and outdoor traffic systems (Castillo et al., 2001; Ng et al., 2013, Di Febbraro et al., 2016), and so on. Among the applications of PNs, the digital internet and the flow of data packets through the net have been often considered in the literature. When switching from the digital world to the physical one, from data packets through the net to physical goods in a logistic system, PNs can be still a valuable modelling formalism and an efficient tool to analyse the performance of the system. Indeed, focusing the attention towards the physical internet, Petri nets seems particularly appropriate to represent the dynamics of π-containers within the PI, as they are able to capture the precedence relations and interactions among events which characterize the facilities and infrastructures (multimodal logistics centres and hubs, transit centres, roads and railways) through which π-containers are delivered. Among the several classes of Petri nets that have been defined in the past, Coloured Petri Nets (CPNs; Jensen and Kristensen, 2009) are especially suitable to model the different kinds of π-containers and the operations required by them and carried out by PI facilities such as π-movers, π-conveyors, π-stores, and so on.

Since their introduction in early ’80s, CPNs have been extensively adopted to model the behavior of complex systems, and several examples can be found in the fields of manufacturing (such as Feldmann and Colombo, 1998; Hsieh and Chen, 1999; Chen and Chen, 2003; Dotoli and Fanti, 2004; Baruwa et al., 2015) and transportation (such as DiCesare et al., 1994; van der Aalst and Odijk, 1995; Dotoli and Fanti, 2006; Huang and Chung, 2008). For what concerns the specific field of logistics, the use of CPNs to model and analyse logistic system has been considered since early 1990s (van der Aalst, 1992) and a review of Petri net-based approaches (including those relevant to CPNs) for logistic system has been proposed in Chen et al. (2006). In van der Vorst et al. (2000), a generic food supply chain is considered and the timed CPN defined by the authors is used to simulate the model; the simulation of a logistic and manufacturing system with CPNs is considered also in Piera et al. (2004) with the objective of optimizing the performance of the system, whereas in Hanafi et al. (2007) the formalism of Fuzzy CPNs is employed to forecast the volumes of returns in a reverse logistics scenario. In Gallash et al. (2008) a CPN is used to model a military logistics system. In Zhang et al. (2009) the coloured Petri net is adopted to configure a supply chain on the basis of customer orders. Optimization through simulation by adopting the CPN formalism is again considered in Narciso et al. (2010), and supply chains are further considered in Zegordi and Davarzani (2012) in which the Petri net model is used to analyse the impact of disruption events. Two recent works are Zhao et al. (2015) and Park et al. (2016), that are both relevant to the simulation of a port logistics system by exploiting a CPN model of the system.

In this paper, CPNs are applied to a multimodal hub in the PI, with the aim of providing a formal model to be used for analysis and optimization purposes. The multimodal hub consists of four areas: a port area at which vessels arrive and depart, a train terminal for rail transportation, a road terminal for truck-to-X transhipment, and a storage area. The rail and the road terminals are equipped with specific devices (e.g., π-composers) in order to facilitate the load and unload operations of trains and trucks; the storage area includes a finite number of π-stores and the four areas are connected through π-conveyors and/or π-movers. In the paper, the storage area and the road terminal are specifically taken into consideration, and the two coloured Petri nets representing an example of π-conveyor and an example of π-sorter/π-composer are described in detail.

The structure of the multimodal hub here considered has been inspired by the Physical Internet Manifesto (Montreuil, 2012) and by its subsequent work about the impact of adoption of the PI on logistics facilities (Montreuil, 2011). In recent years, the interest on the Physical Internet has grown and many works appeared in the literature aimed at formalizing the working principles of items and resources involved in a PI, both from the technological point of view and from the logical/functional one. The formalization of π-containers, their movements and storage within a PI, and the way they can be merged into larger composed containers, have been considered in Landschützer et al. (2015) and Montreuil et al. (2016) that provide several examples of π-containers and also report some results coming from the European project MODULUSHCA; the active role of π-containers within next-generation supply chains is also discussed in Sallez et al. (2016). Instead, for what concerns PI facilities, the physical elements serving as the foundation of the Physical Internet infrastructure have been introduced for the first time in Montreuil et al. (2010), whereas the functional design of specific kinds of hubs and transit centres have been taken into account in Ballot et al. (2014), Meller et al. (2014), and Montreuil et al. (2014).

The proposed model can be used, in general, both for analysing the structural properties of the system (first of all, to check if the system is deadlock-free or not) and to optimize some parameters of the system (via simulation, that is, by using the PN as an intermediate model between real world and simulation tools). However, it is worth observing that the primary aim of this paper is to show the applicability of the Petri net formalism to the Physical Internet paradigm, without being exhaustive. Besides, according to our best knowledge, this is the first work on the application of Petri nets to the Physical Internet.

2 The model of the multimodal hub

The multimodal hub consists of π-nodes and π-movers which interact with the objective of optimizing logistics operations on the standardized π-containers. The part of the multimodal hub considered in this paper is illustrated in Figure 1.

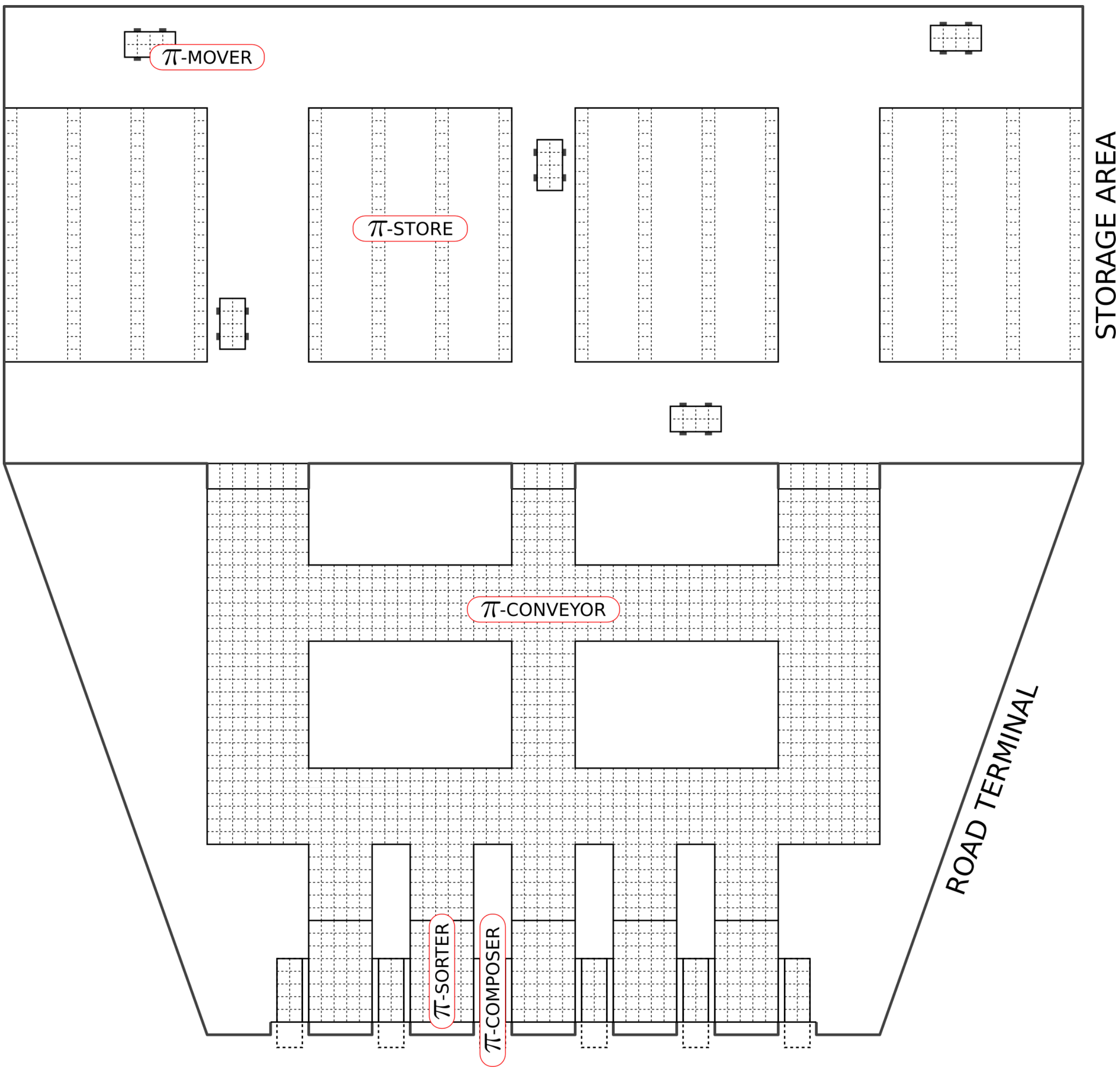
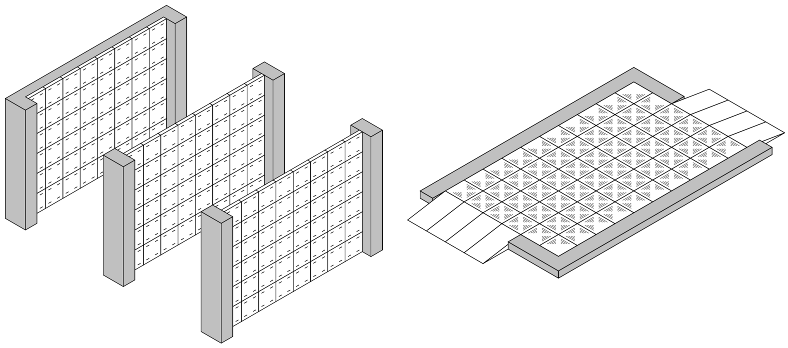


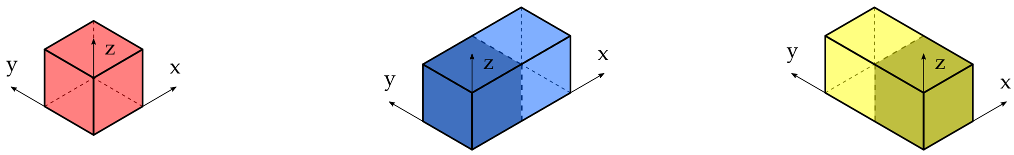
Figure 1: Sketch of the system layout.



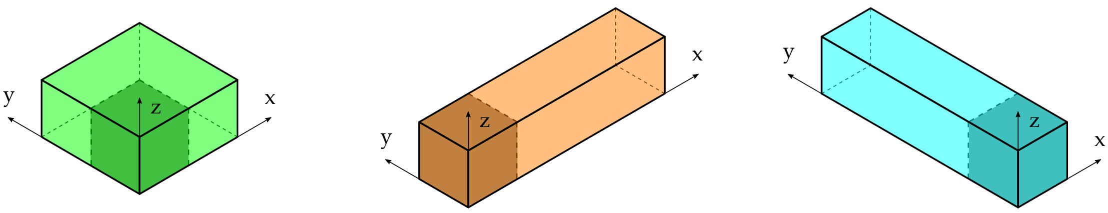
(a) (b) (c)

Figure 2: Details of the physical model of (a) π-store, (b) π-conveyor, and (c) π-sorter.

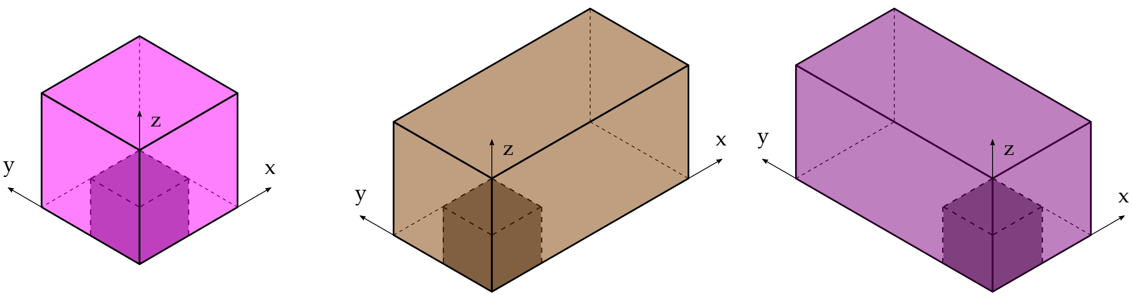
The storage area consists of some snapping π-stores which are served by a set of automated-guided π-movers which handle the π-containers from the storage area to the road terminal; an example of π-store here considered is in Figure 2(a). Within the road terminal items are handled with a π-conveyor which moves the π-containers from the border of the storage area to the π-sorters which manage the π-containers of the various sizes so that they can be aggregated by means of the π-composer and delivered by road with a π-carrier. An example of a section of the π-conveyor is reported in Figure 2(b) whereas Figure 2(c) illustrates the physical part including a π-sorter and two π-composers. The section of the π-conveyor and the π-sorter with the two π-composers will be represented with Coloured Petri Nets in Section 3.



(a) size 111: 2.4x2.4x2.4 m^3 (b) size 211: 4.8x2.4x2.4 m^3 (c) size 121: 2.4x4.8x2.4 m^3



(d) size 221: 4.8x4.8x2.4 m^3 (e) size 411: 9.6x2.4x2.4 m^3 (f) size 141: 2.4x4.8x2.4 m^3



(g) size 222: 4.8x4.8x4.8 m^3 (h) size 422: 9.6x4.8x4.8 m^3 (i) size 242: 4.8x9.6x4.8 m^3

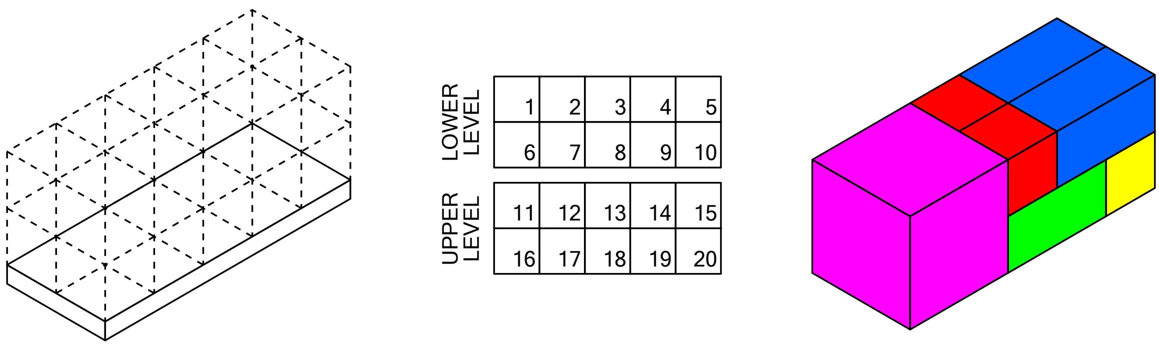
Figure 3: Types of π-containers.

In this paper, 9 sizes of π-containers are taken into consideration; they are illustrated in Figure 3. It can be assumed that smaller π-containers (such as those with sizes ranging from 0.12 m to 0.6 m), if present, are suitably encapsulated within the π-container of Figure 3(a) that, in the following, will be often referred to as "π-container of unitary size" or "basic π-container" (instead, for what concern larger π-containers, the proposed approach can be easily generalized in order to take into account π-containers of any standardized size). Moreover, it can be observed that the second and the third type of π-containers (provided in Figures 3(b) and 3(c), respectively), the fifth and the sixth (Figures 3(e) and 3(f)), and the eighth and the ninth (3(h) and 3(i)) have the same size; in this work, they are considered as different in order to help the representation of the system dynamics with Petri nets. Finally, the concept of "π-core" is here introduced; basically, the π-core is the "dark cube" of unitary size which is inside the π-containers of Figures 3(b)÷3(i); as illustrated, the π-core is conventionally located at the origin of the 3-dimensional Cartesian axes which identify the size and orientation of a π-container.

2.1 π-core

In the new logistics facilities and material handling systems that are compatible with the PI paradigm, many π-nodes and π-movers will be built exploiting the concept of π-cell, that is, a "structure" of unitary size that, for example, can help π-containers to move along the π-conveyors or can allow π-containers to hold on to the racks of a π-store. It is evident that all the resources depicted in Figure 1 and Figures 2(a), 2(b), and 2(c) (π-stores, π-movers, π-conveyor, π-sorters and π-composers) are made of several cells. When a π-container uses one of such resources, it occupies one or more cells: a π-container of unitary size occupies a single cell whereas larger π-containers occupy 2, 4, or 8 cells, depending on their size.

In the proposed CPN model[1], the concept of π-core allows representing a π-container with a single coloured token, keeping in this way the model simple. As it will be shown in the following section, each π-cell is represented by a place and a coloured token inside the place means that the π-core of a π-container with size specified by the colour of the token is occupying the π-cell.



(a) (b) (c)

Figure 4: π-containers (composed): (a) structure, (b) logical scheme, and (c) example.

2.2 π-container (composed)

Another fundamental aspect in the Physical Internet paradigm is the possibility of composing containers of standard size by suitably attaching π-containers of different sizes. In this paper, it is assumed that the trucks leaving the road terminal carry containers whose size is 12x4.8x4.8 m^3 which corresponds to the composition of 20 π-containers of unitary size or a lower number of larger π-containers. In Figure 4(c) a container composed by one π-container of size 222, one of size 221, one of size 121, two of size 111, and two of size 211 is illustrated.

[1] It is assumed that the reader has a basic knowledge of the Petri net formalism; otherwise, he/she can refer to Murata (1989) for a detailed presentation of PN's definitions, rules, and properties. Instead, for what concerns Coloured Petri Nets, the reader can refer to Jensen and Kristensen (2009). In any case, the definition of the adopted class of CPNs is given in Section 3.

In the proposed CPN representation, the composed container will be modelled with a single coloured token whose colour is a vector of elements describing the way the container is composed. Such a vector contains 20 elements and its i-th element correspond to the i-th cell of the logical scheme illustrated in Figure 4(b). Each element of such a vector can be either 0 or one of the allowed sizes (namely, 111, 211, 121, and so on): in the latter case, it means that a π-container of the specified size has its π-core in the i-th cell. As an example, the composed container illustrated in figure 4(c) is represented by the following vector:

$$(0,0,0,0,0,222,0,221,0,121,0,0,111,211,0,0,0,111,211,0)$$

that corresponds to the scheme

0	0	0	0	0
222	0	221	0	121

LOWER LEVEL (cells #1÷#10)

0	0	111	211	0
0	0	111	211	0

UPPER LEVEL (cells #11÷#20)

in which the magenta π-container of size 222 has its π-core in cell #6 (lower level), the green π-container of size 221 has its π-core in cell #8 (lower level), the yellow π-container of size 121 has its π-core in cell #10 (lower level), the two red π-containers of size 111 have their π-core in cells #13 and #18 (upper level), and the two blue π-containers of size 211 have their π-core in cells #14 and #19 (upper level).

In connection with composed containers, it is also necessary to know the sequence of loads of the single π-containers carried out by the π-composer. Such a piece of information is used in the CPN representation to define the guard functions of some transitions in the CPN which models the π-composer, and can be defined as an ordered sequence of integer numbers in the range $[1,20]$; in this way, a number i in the j-th position of the sequence means that the π-container whose π-core has to be placed in the i-th cell of the composed container must be the j-th in the loading sequence. As an example, the sequence of loads for the composed container illustrated in Figure 4(c) is $(6,8,10,18,13,19,14)$. It is worth noting that, all possible loading sequences can be a-priori defined on the basis of the allowed structures of composed containers.

3 CPN representation

In this paper, two Coloured Petri Nets are proposed with the aim of showing the applicability of such formalism to an example of multimodal hub compatible with the Physical Internet paradigm. The first net is relative to a generic section of the π-conveyor whereas the second net models one of the available π-sorter/π-composer. In both cases, the behaviour of the CPN is illustrated through an example. The adopted class of CPN is the following.

Definition 1 (Jensen and Kristensen, 2009) – A (non-hierarchical) Coloured Petri Net is a nine-tuple $CPN = (P, T, \mathcal{A}, \Sigma, \mathcal{V}, \mathcal{C}, G, E, I)$, where:

1. P is a finite set of places;
2. T is a finite set of transitions ($P \cap T = \emptyset$);
3. $\mathcal{A} \subseteq P \times T \cup T \times P$ is a set of directed arcs;
4. Σ is a finite set of non-empty colour sets;
5. $\mathcal{V}$ is a finite set of typed variables such that $\mathrm{Type}[v] \in \Sigma$ for all variables $v \in \mathcal{V}$;
6. $\mathcal{C}: P \to \Sigma$ is a colour set function that assigns a colour set to each place;
7. $G: T \to \mathrm{Expr}_V$ is a guard function that assigns a guard to each transition t such that $\mathrm{Type}[G(t)] = \mathrm{Bool}$ (true/false);
8. $E: A \to \mathrm{Expr}_V$ is an arc expression function that assigns an arc expression to each arc a such that $\mathrm{Type}[E(a)] = \mathcal{C}(\mathrm{p})_{MS}$, where p is the place connected to the arc a;
9. $I: P \to \mathrm{Expr}_\emptyset$ is an initialization function that assigns an initialization expression to each place p such that $\mathrm{Type}[I(p)] = \mathcal{C}(\mathrm{p})_{MS}$.

Some guard functions and arc expressions, as well as the set of typed variables and the colour set functions, will be defined in subsections 3.1 and 3.2 for the coloured Petri nets representing the π-conveyor and the π-sorter/π-composer. Instead, for what concern the colours sets, they are relevant to the size of π-containers and to the structure of composed containers leaving the road terminal; then,

$$\Sigma = \{\mathrm{BSIZE}, \mathrm{BSIZE0}, \mathrm{STRUCT}\} \tag{1}$$

with

$$\mathrm{BSIZE} = \{111, 211, 121, 221, 411, 141, 222, 422, 242\} \tag{2}$$

$$\mathrm{BSIZE0} = \mathrm{BSIZE} \cup \{0\} \tag{3}$$

$$\mathrm{STRUCT} = BSIZE0^{20} = \mathrm{BSIZE0} \times \mathrm{BSIZE0} \times \mathrm{BSIZE0} \times \ldots \times \mathrm{BSIZE0} \times \mathrm{BSIZE0} \text{ (20 times)} \tag{4}$$

3.1 π-conveyor

The considered section of the π-conveyor is illustrated in Figures 5 (physical model) and 6 (logical representation). The flow of π-containers is both from left to right and vice-versa but they can also change position by moving up and down (in order to optimize the flow of goods on the conveyor). The entry points are located in the example in the top left part and in the bottom right part areas, but any border cell can be a cell of entrance to the conveyor. In any case, apart from borders, π-containers can move freely when handled by the π-conveyor.

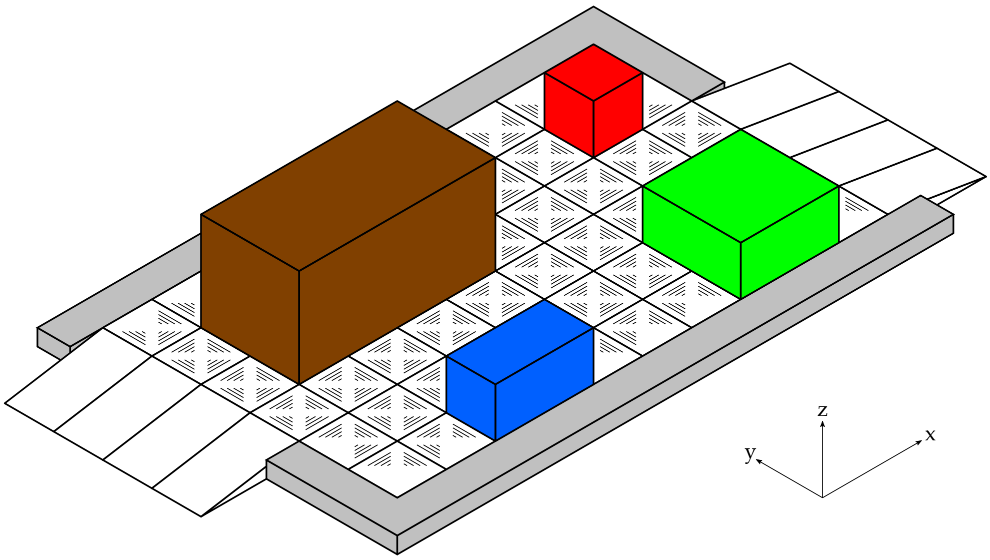


Figure 5: Physical model of a section of the π-conveyor.

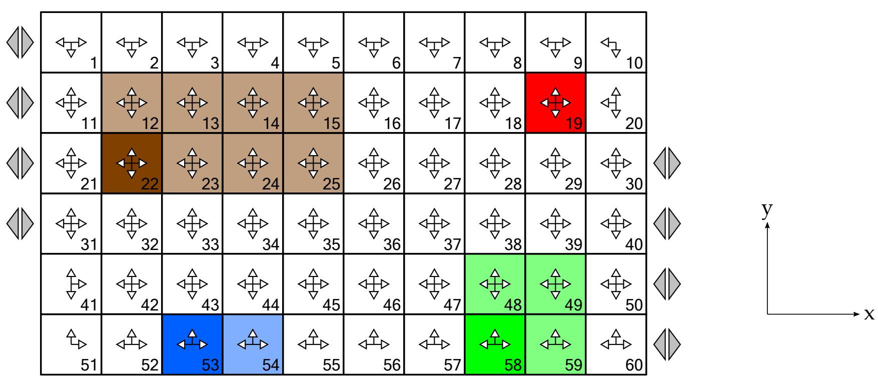


Figure 6: Logical representation of a section of the π-conveyor.

Such a model is represented by means of the coloured Petri net illustrated in Figure 7. Each place represents a cell of the π-conveyor and a token in a place means that a π-container has its π-core over the cell. It is obvious that, depending on the type (size) of the π-container, other adjacent cells are occupied by the π-container, even if the corresponding places have no token. As an example, the brown π-container in Figure 5 has its π-core over the cell #22 but it occupies also cells #12÷15 and #23÷25; in the CPN, a token with colour 422 is inside place p_{22} but also places p_{12}÷p_{15} and p_{23}÷p_{25} are "virtually marked" in the sense that no token (representing a different π-container) can enter such places. This is ruled by the guard functions associated with the transitions of the CPN. Each transition models the movement of the π-core of a π-container to one of the four adjacent cells (less than four in the border cells), that is, towards east, north, west, south. The token game models the movements of π-containers over the π-conveyor.

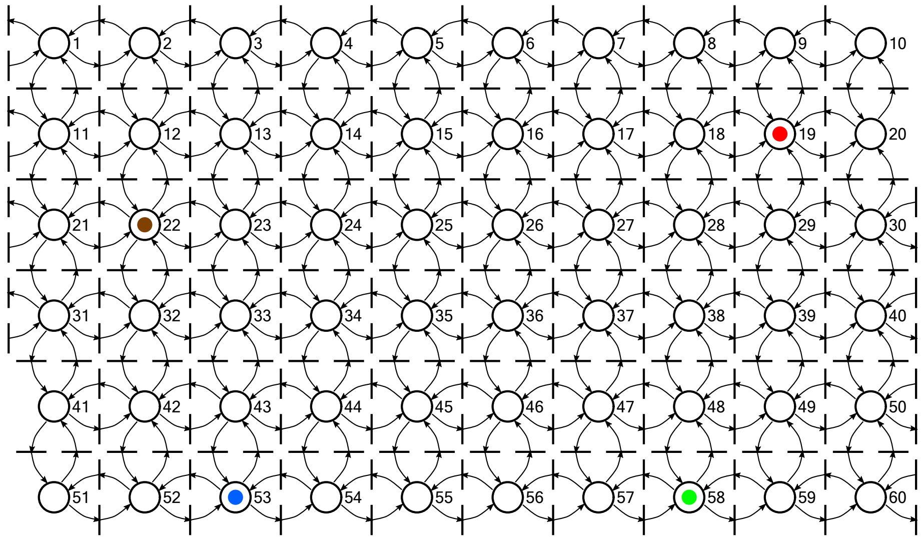


Figure 7: CPN representing the section of the π-conveyor.

The colour set functions, the set of typed variables, the arc expressions, and the guard functions are reported in the following.

$$\mathcal{C}(p_h) = \text{BSIZE}\,, \forall h = 1, \dots, 60 \tag{5}$$

$$\mathcal{V} = \{b_h\text{:BSIZE};\, h = 1, \dots, 60\} \tag{6}$$

$$\text{E}(p_h, t_{h-k}) = 1'b_h\,, \forall h, k = 1, \dots, 60 \tag{7}$$

$$\text{E}(t_{h-k}, p_k) = 1'b_h\,, \forall h, k = 1, \dots, 60 \tag{8}$$

$$\text{G}(t_{h-k}) = [(b_h = 111) \wedge (\text{C}_{h-k-111})] \vee [(b_h = 211) \wedge (\text{C}_{h-k-211})] \vee [(b_h = 121) \wedge (\text{C}_{h-k-121})] \vee [(b_h = 221) \wedge (\text{C}_{h-k-221})] \vee [(b_h = 411) \wedge (\text{C}_{h-k-411})] \vee [(b_h = 141) \wedge (\text{C}_{h-k-141})] \vee [(b_h = 222) \wedge (\text{C}_{h-k-222})] \vee [(b_h = 422) \wedge (\text{C}_{h-k-422})] \vee [(b_h = 242) \wedge (\text{C}_{h-k-242})]\,, \forall h, k = 1, \dots, 60 \tag{9}$$

The guard function defined in (9) shows that the firing of a transition depends on the colour of the token in the corresponding input place. In fact, it is evident that the possibility of moving a π-container whose π-core is over a certain cell changes with the size of the π-container itself. As an example, the brown π-container with the π-core in cell #22 can move eastbound if cells #16 and #26 are free and can move southbound if cells #32, #33, #34, and #35 are free; instead, in the case that the π-container with the π-core in cell #22 is blue (with size 211), then it can move eastbound if cell #24 is free and can move southbound if cells #32 and #33 are free. In this connection, C_{h-k-s} is the Boolean condition to make transition t_{h-k} fireable when a token of colour s is within place p_h.

(a) (b)

Figure 8: Detail of the CPN representing the section of the π-conveyor: movement from cell #22 to cells #23 (eastbound) and #32 (southbound).

In the following, some of these Boolean conditions are reported. They are relative to the places p_{22}, p_{23}, p_{32} and transitions t_{22-23}, t_{22-32}, as illustrated in Figure 8(a) and 8(b) where, for the sake of brevity only two colours (sizes) are considered: 422 (brown π-container) and 211 (blue π-container). The condition $\mathrm{C}_{22-23-422}$ take into consideration all the cases in which cells #16 and #26 are free, that is, all the cases in which places p_{16} and p_{26} are neither marked nor "virtually marked". $\mathrm{C}_{22-32-422}$ is relative to the move towards south and then it considers all the cases in which places p_{32}, p_{33}, p_{34}, and p_{35} are neither marked nor "virtually marked". Conditions $\mathrm{C}_{22-23-211}$ and $\mathrm{C}_{22-32-211}$ report the cases in which p_{24}, and p_{32}, p_{33}, respectively, are neither marked nor "virtually marked".

$$\mathrm{C}_{22-23-422} = [(\mathrm{b}_{16} \neq 111)\vee(\mathrm{b}_{16} \neq 121)\vee(\mathrm{b}_{16} \neq 211)\vee(\mathrm{b}_{16} \neq 221)\vee(\mathrm{b}_{16} \neq 411)\vee(\mathrm{b}_{16} \neq 222)\vee(\mathrm{b}_{16} \neq 422)]\wedge[(\mathrm{b}_{26} \neq 111)\vee(\mathrm{b}_{26} \neq 121)\vee(\mathrm{b}_{26} \neq 211)\vee(\mathrm{b}_{26} \neq 221)\vee(\mathrm{b}_{26} \neq 411)\vee(\mathrm{b}_{26} \neq 222)\vee(\mathrm{b}_{26} \neq 422)]\wedge[(\mathrm{b}_{36} \neq 121)\vee(\mathrm{b}_{36} \neq 221)\vee(\mathrm{b}_{36} \neq 141)\vee(\mathrm{b}_{36} \neq 222)\vee(\mathrm{b}_{36} \neq 422)\vee(\mathrm{b}_{36} \neq 242)]\wedge[(\mathrm{b}_{46} \neq 141)\vee(\mathrm{b}_{46} \neq 242)]\wedge[(\mathrm{b}_{56} \neq 141)\vee(\mathrm{b}_{56} \neq 242)] \quad (10)$$

$$\mathrm{C}_{22-32-422} = [(\mathrm{b}_{31} \neq 211)\vee(\mathrm{b}_{31} \neq 411)]\wedge[(\mathrm{b}_{32} \neq 111)\vee(\mathrm{b}_{32} \neq 211)\vee(\mathrm{b}_{32} \neq 411)]\wedge[(\mathrm{b}_{33} \neq 111)\vee(\mathrm{b}_{33} \neq 211)\vee(\mathrm{b}_{33} \neq 411)]\wedge[(\mathrm{b}_{34} \neq 111)\vee(\mathrm{b}_{34} \neq 211)\vee(\mathrm{b}_{34} \neq 411)]\wedge[(\mathrm{b}_{35} \neq 111)\vee(\mathrm{b}_{35} \neq 211)\vee(\mathrm{b}_{35} \neq 411)]\wedge[(\mathrm{b}_{41} \neq 221)\vee(\mathrm{b}_{41} \neq 222)\vee(\mathrm{b}_{41} \neq 422)]\wedge[(\mathrm{b}_{42} \neq 121)\vee(\mathrm{b}_{42} \neq 221)\vee(\mathrm{b}_{42} \neq 222)\vee(\mathrm{b}_{42} \neq 422)]\wedge[(\mathrm{b}_{43} \neq 121)\vee(\mathrm{b}_{43} \neq 221)\vee(\mathrm{b}_{43} \neq 222)\vee(\mathrm{b}_{43} \neq 422)]\wedge[(\mathrm{b}_{44} \neq 121)\vee(\mathrm{b}_{44} \neq 221)\vee(\mathrm{b}_{44} \neq 222)\vee(\mathrm{b}_{44} \neq 422)]\wedge[(\mathrm{b}_{45} \neq 121)\vee(\mathrm{b}_{45} \neq 221)\vee(\mathrm{b}_{45} \neq 222)\vee(\mathrm{b}_{45} \neq 422)] \quad (11)$$

$$\mathrm{C}_{22-23-211} = [(\mathrm{b}_{24} \neq 111)\vee(\mathrm{b}_{24} \neq 121)\vee(\mathrm{b}_{24} \neq 211)\vee(\mathrm{b}_{24} \neq 221)\vee(\mathrm{b}_{24} \neq 411)\vee(\mathrm{b}_{24} \neq 222)\vee(\mathrm{b}_{24} \neq 422)]\wedge[(\mathrm{b}_{34} \neq 121)\vee(\mathrm{b}_{34} \neq 221)\vee(\mathrm{b}_{34} \neq 141)\vee(\mathrm{b}_{34} \neq 222)\vee(\mathrm{b}_{34} \neq 422)\vee(\mathrm{b}_{34} \neq 242)]\wedge[(\mathrm{b}_{44} \neq 141)\vee(\mathrm{b}_{44} \neq 242)]\wedge[(\mathrm{b}_{54} \neq 141)\vee(\mathrm{b}_{54} \neq 242)] \quad (12)$$

$$\mathrm{C}_{22-32-211} = [(\mathrm{b}_{31} \neq 211)\vee(\mathrm{b}_{31} \neq 411)]\wedge[(\mathrm{b}_{32} \neq 111)\vee(\mathrm{b}_{32} \neq 211)\vee(\mathrm{b}_{32} \neq 411)]\wedge[(\mathrm{b}_{33} \neq 111)\vee(\mathrm{b}_{33} \neq 211)\vee(\mathrm{b}_{33} \neq 411)]\wedge[(\mathrm{b}_{41} \neq 221)\vee(\mathrm{b}_{41} \neq 222)\vee(\mathrm{b}_{41} \neq 422)]\wedge[(\mathrm{b}_{42} \neq 121)\vee(\mathrm{b}_{42} \neq 221)\vee(\mathrm{b}_{42} \neq 222)\vee(\mathrm{b}_{42} \neq 422)]\wedge[(\mathrm{b}_{43} \neq 121)\vee(\mathrm{b}_{43} \neq 221)\vee(\mathrm{b}_{43} \neq 222)\vee(\mathrm{b}_{43} \neq 422)] \quad (13)$$

It is worth finally noting that C_{h-k-s} is set to "false" (or, equivalently, $\mathrm{C}_{h-k-s} = 0$) when a π-container of size s cannot be in correspondence of the h-th cell; for example, in the cell #22 it is not possible to have π-containers of size 141 and 242, and then $\mathrm{C}_{22-k-141} = 0$ and $\mathrm{C}_{22-k-242} = 0$ for any $k = 12,21,23,32$.

With the considered guard functions, all possible physical conflicts between π-containers are prevented, and thus the movements of coloured tokens within the CPN actually correspond to feasible movements of π-containers over the π-conveyor.

3.2 π-sorter/π-composer

The material handling system for the composition of containers of standard size to be delivered by road with trucks is here considered and modelled with a coloured Petri net. The structure illustrated in Figure 9 is taken into consideration: it consists of a π-sorter which handles π-containers and puts them in the correct position at the correct time in order to load them into one of the two π-composers (A and B) that are at opposite sides of the π-sorter[2]. The π-containers are loaded by following a strict order (loading sequence), and the π-container to be loaded can start its loading operation only when it has its π-core over the right cell. As an example, the seven π-containers that are included in Figure 9 have to be composed by the π-composer B (the one on the right side) to form the structure illustrated in Figure 4(c); this can be done by following the loading sequence reported in Table 1.

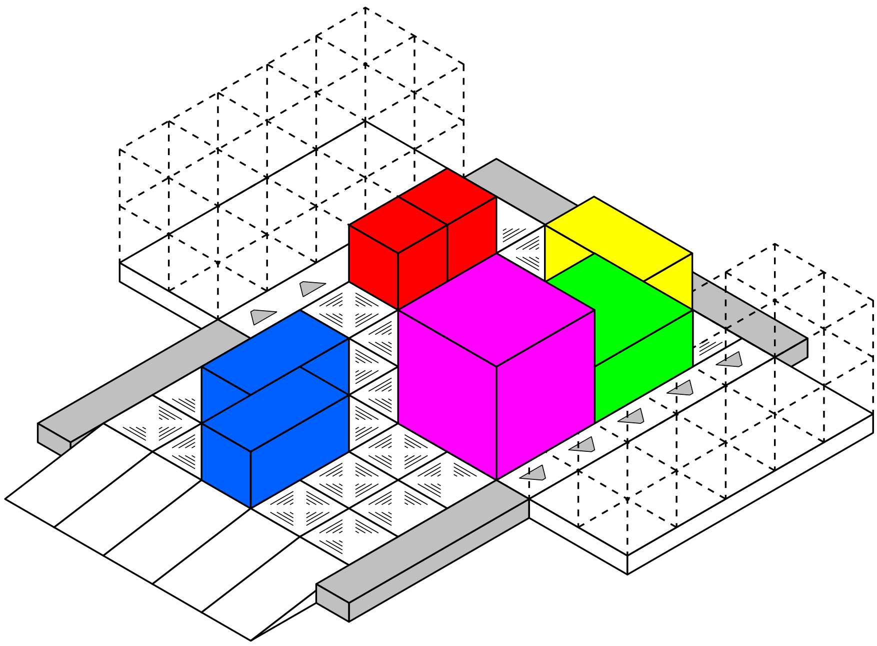


Figure 9: Physical model of a π-sorter with two π-composers.

Table 1: Loading sequence of the π-containers in Figure 9.

Sequence	π-container	π-core at start (cell #)	π-core at end (cell #)
1st	222 (magenta)	36	B6
2nd	221 (green)	38	B8
3rd	121 (yellow)	40	B10
4th	111 (red)	38	B18
5th	111 (red)	38	B13
6th	211 (blue)	39	B19
7th	211 (blue)	39	B14

[2] The problem of lifting the π-containers to the upper level of the π-composer is not addressed here, being out of the scope of this paper; as a matter of fact, it is here assumed that suitable handling systems exist and are able to move π-containers to any position of the π-composer.

In the loading sequence, also the destination cell for the π-core of the π-container which is loaded is reported in Table 1; letter B, obviously, refers to π-composer B, whereas the number correspond to the 3-dimensional cell structure defined in Figures 4(a) and 4(b). The final state of the material handling system π-sorter/π-composers is given in Figure 10.

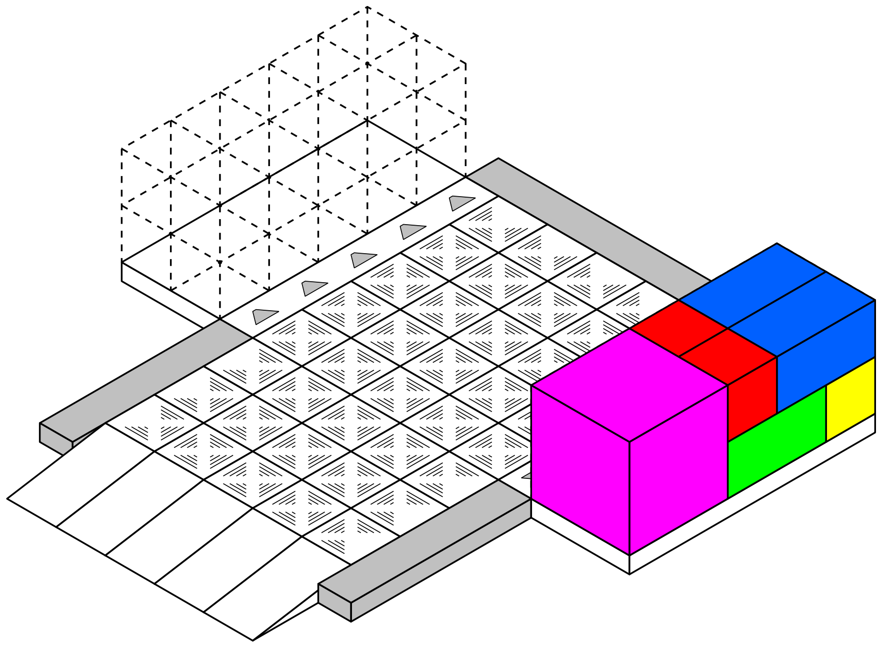


Figure 10: π-sorter/π-composer after the composition of a container.

Such a model is represented by means of the coloured Petri net illustrated in Figure 11. Places from p_1 to p_{40} represent the cells of the π-sorter and the transitions between these places model the movements of the π-core of π-containers from one cell to another; as a matter of fact, this part of the CPN has a mode of operation which is analogous to that of the CPN representing the π-conveyor.

The composition of a container is modelled with places from p_{41} to p_{60} and p_A (for the π-composer A) and from p_{61} to p_{60} and p_B (π-composer B). Places p_{41}÷p_{60} and p_{61}÷p_{80} contain coloured tokens that represents single π-containers placed in the 3-dimensional structure illustrated in Figures 4(a), 9 and 10. As illustrated in Figure 11, such places receive tokens from places p_4÷p_8 and p_{36}÷p_{40}, respectively, in accordance with some firing rules which are derived from the loading sequence of the container to be composed. Instead places p_A and p_B contain tokens whose colour is a structure defining the way a container is composed; such tokens are created by firing transitions t_A and t_B, respectively, which remove all tokens from places p_{41}÷p_{60} and p_{61}÷p_{80}; such firings are ruled by suitable guard functions which prevent the firing in the case the container is not composed appropriately.

The colour set functions, the set of typed variables, the arc expressions, and the guard functions are reported in the following. In equations (27)÷(30), f_{LS} and g_{LS} are two functions which express the fact that the guard functions associated with the transitions included in the part of the CPN relevant to π-composers A and B are defined in accordance with the loading sequences of the containers to be composed (an example of such functions is provided later).

$$\mathcal{C}(p_h) = \mathrm{BSIZE}\,, \forall h = 1, \ldots, 80 \quad (14)$$

$$\mathcal{C}(p_l) = \mathrm{STRUCT}\,, \forall l = \mathrm{A,B} \quad (15)$$

$$\mathcal{V} = \{b_h{:}\mathrm{BSIZE}; h = 1, \ldots, 80\} \quad (16)$$

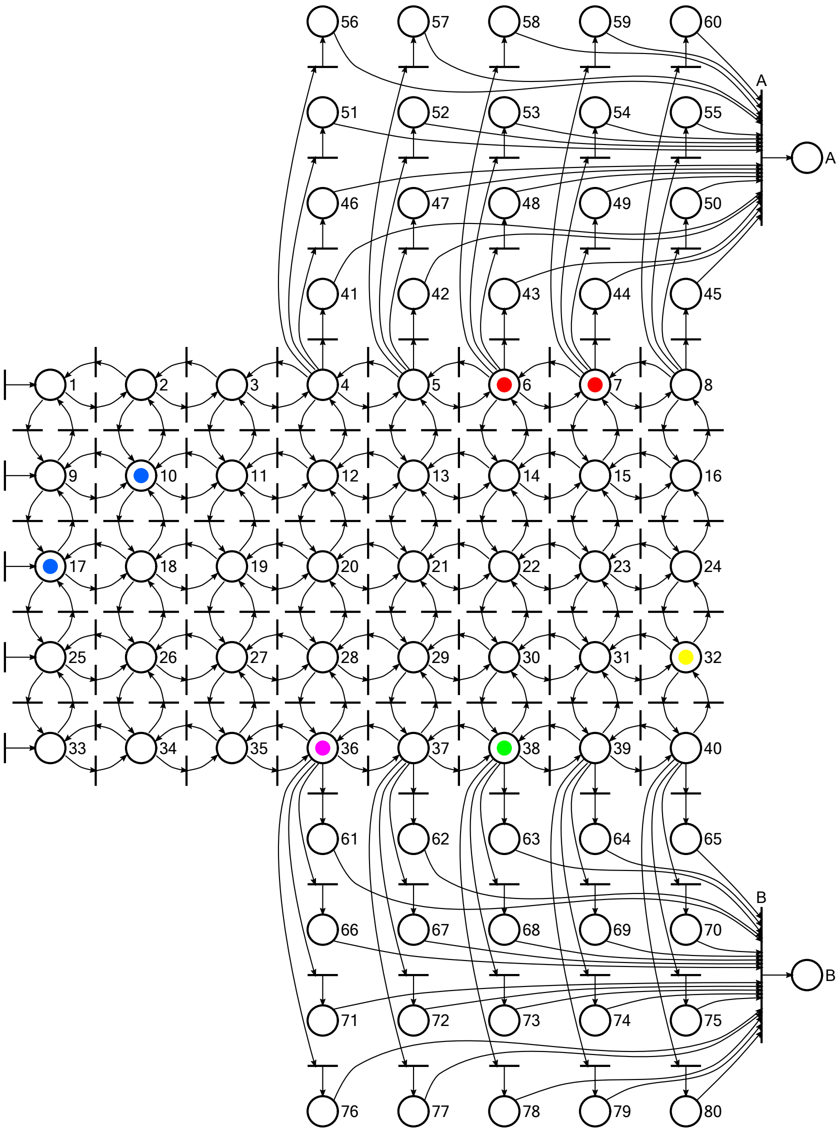


Figure 11: CPN representing the π-sorter with two π-composers.

$$\mathrm{E}(p_h, t_{h-k}) = 1'b_h \text{ , } \forall h, k = 1, \dots, 40 \tag{17}$$

$$\mathrm{E}(t_{h-k}, p_k) = 1'b_h \text{ , } \forall h, k = 1, \dots, 40 \tag{18}$$

$$\mathrm{E}(p_h, t_{\mathrm{A}k}) = 1'b_h \text{ , } \forall h, k\colon (p_h, t_{\mathrm{A}k}) \in \mathcal{A} \text{ , } h \in \{4, \dots, 8\} \text{ , } k \in \{41, \dots, 60\} \tag{19}$$

$$\mathrm{E}(t_{\mathrm{A}k}, p_k) = 1'b_h \text{ , } \forall h, k\colon (p_h, t_{\mathrm{A}k}) \in \mathcal{A} \text{ , } h \in \{4, \dots, 8\} \text{ , } k \in \{41, \dots, 60\} \tag{20}$$

$$\mathrm{E}(p_h, t_{\mathrm{B}k}) = 1'b_h \text{ , } \forall h, k\colon (p_h, t_{\mathrm{B}k}) \in \mathcal{A} \text{ , } h \in \{36, \dots, 40\} \text{ , } k \in \{61, \dots, 80\} \tag{21}$$

$$\mathrm{E}(t_{\mathrm{B}k}, p_k) = 1'b_h \text{ , } \forall h, k\colon (p_h, t_{\mathrm{B}k}) \in \mathcal{A} \text{ , } h \in \{36, \dots, 40\} \text{ , } k \in \{61, \dots, 80\} \tag{22}$$

$$\mathrm{E}(p_h, t_l) = 1'b_h \text{ , } \forall h = 41, \dots, 60 \text{ when } l = \mathrm{A} \text{ , } \forall h = 61, \dots, 80 \text{ when } l = \mathrm{B} \tag{23}$$

$$\mathrm{E}(t_\mathrm{A}, p_\mathrm{A}) = 1'(b_{41}, \dots, b_{60}) \tag{24}$$

$$\mathrm{E}(t_\mathrm{B}, p_\mathrm{B}) = 1'(b_{61}, \dots, b_{80}) \tag{25}$$

$$\mathrm{G}(t_{h-k}) = [(b_h = 111) \wedge (\mathrm{C}_{h-k-111})] \vee [(b_h = 211) \wedge (\mathrm{C}_{h-k-211})] \vee [(b_h = 121) \wedge (\mathrm{C}_{h-k-121})] \vee [(b_h = 221) \wedge (\mathrm{C}_{h-k-221})] \vee [(b_h = 411) \wedge (\mathrm{C}_{h-k-411})] \vee [(b_h = 141) \wedge (\mathrm{C}_{h-k-141})] \vee [(b_h = 222) \wedge (\mathrm{C}_{h-k-222})] \vee [(b_h = 422) \wedge (\mathrm{C}_{h-k-422})] \vee [(b_h = 242) \wedge (\mathrm{C}_{h-k-242})] \text{ , } \forall h, k = 1, \dots, 60 \tag{26}$$

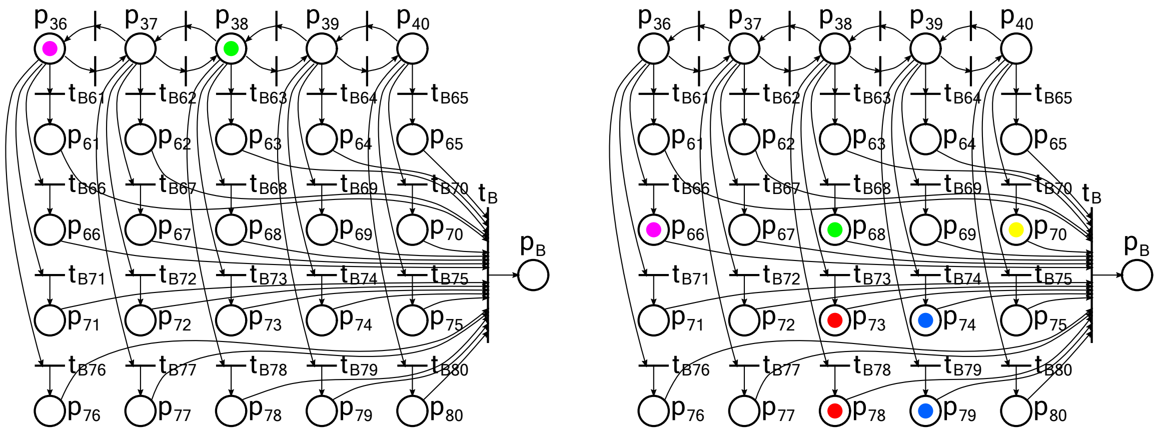
$$\mathrm{G}(t_{\mathrm{A}k}) = f_{\mathrm{LS}}(b_4, \dots, b_8, b_{41}, \dots, b_{60}) \tag{27}$$

$$\mathrm{G}(t_{\mathrm{B}k}) = f_{\mathrm{LS}}(b_{36}, \dots, b_{40}, b_{61}, \dots, b_{80}) \tag{28}$$

$$\mathrm{G}(t_{\mathrm{A}}) = g_{\mathrm{LS}}(b_{41}, \dots, b_{60}) \tag{29}$$

$$\mathrm{G}(t_{\mathrm{B}}) = g_{\mathrm{LS}}(b_{61}, \dots, b_{80}) \tag{30}$$

Consider now the part of the CPN representing the π-sorter/π-composers which is relative to the π-composer B (it is illustrated in Figure 12), and assume that the container to be composed has the structure illustrated in Figure 4(c). The marking to be reached, starting from the one in Figures 11 and 12(a), is the marking illustrated in Figure 12(b) which correspond to the Petri net representation of the vector (0,0,0,0,0,222,0,221,0,121,0,0,111,211,0,0,0,111,211,0) that is adopted to describe the composed container.



(a) (b)

Figure 12: Detail of the CPN representing the π-sorter with two π-composers: (a) start of composition on π-composer B, and (b) end of composition on π-composer B (final marking).

To reach the final marking in Figure 12(b), coloured tokens representing the seven π-containers of sizes 222, 221, 121, 111, and 211, must reach places p_{36} (the one of size 222), p_{38} (the one of size 221 and the two of size 111), p_{39} (the two of size 211), and p_{40} (the one of size 121) through some moving operations on the π-sorter. From such places tokens are inserted into places p_{66}, p_{68}, p_{70}, p_{73}, p_{74}, p_{78}, p_{79} by firing transitions $t_{\mathrm{B}66}$, $t_{\mathrm{B}68}$, $t_{\mathrm{B}70}$, $t_{\mathrm{B}73}$, $t_{\mathrm{B}74}$, $t_{\mathrm{B}78}$, $t_{\mathrm{B}79}$, respectively. However, such firings must follow a strict order corresponding to the loading sequence reported in Table 1. Then, transitions must fire with the sequence $t_{\mathrm{B}66}$, $t_{\mathrm{B}68}$, $t_{\mathrm{B}70}$, $t_{\mathrm{B}78}$, $t_{\mathrm{B}73}$, $t_{\mathrm{B}79}$, $t_{\mathrm{B}74}$, and this is actualized by means of the following guard functions (which also check the presence of the right π-container in places p_{36}÷p_{40}).

$$\mathrm{G}(t_{\mathrm{B}66}) = (b_{36} = 222) \tag{31}$$

$$\mathrm{G}(t_{\mathrm{B}68}) = (b_{38} = 221) \wedge (b_{66} = 222) \tag{32}$$

$$\mathrm{G}(t_{\mathrm{B}70}) = (b_{40} = 121) \wedge (b_{68} = 221) \tag{33}$$

$$\mathrm{G}(t_{\mathrm{B}78}) = (b_{38} = 111) \wedge (b_{70} = 121) \tag{34}$$

$$\mathrm{G}(t_{\mathrm{B}73}) = (b_{38} = 111) \wedge (b_{78} = 111) \tag{35}$$

$$\mathrm{G}(t_{\mathrm{B}79}) = (b_{39} = 211) \wedge (b_{73} = 111) \tag{36}$$

$$\mathrm{G}(t_{\mathrm{B}74}) = (b_{39} = 211) \wedge (b_{79} = 211) \tag{37}$$

$$\mathrm{G}(t_{\mathrm{B}k}) = 0\,,\ k = 61,62,63,64,65,67,69,71,72,75,76,77,80 \tag{38}$$

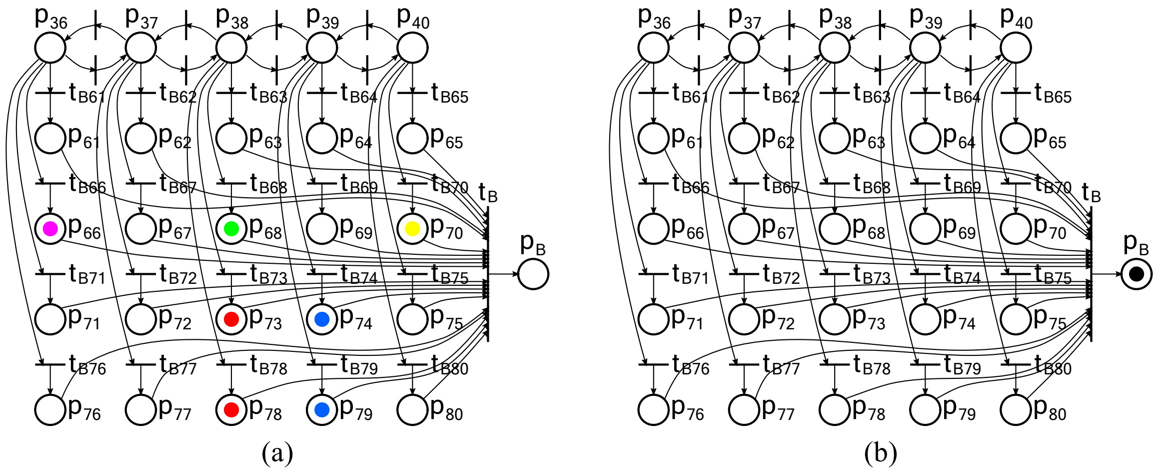


Figure 13: Detail of the CPN representing the π-sorter with two π-composers: (a) end of composition on π-composer B (final marking), and (b) consolidation of the composed container.

Once reached the final marking for what concerns places $p_{61} \div p_{80}$, it is possible to consolidate the composed container by firing transition t_B (see Figure 13), which is now enabled in accordance with the logical condition defined by the guard function (39).

$$\mathrm{G}(t_{\mathrm{B}}) = (b_{61} = 0) \wedge (b_{62} = 0) \wedge (b_{63} = 0) \wedge (b_{64} = 0) \wedge (b_{65} = 0) \wedge (b_{66} = 222) \wedge (b_{67} = 0) \wedge (b_{68} = 221) \wedge (b_{69} = 0) \wedge (b_{70} = 121) \wedge (b_{71} = 0) \wedge (b_{72} = 0) \wedge (b_{73} = 111) \wedge (b_{74} = 211) \wedge (b_{75} = 0) \wedge (b_{76} = 0) \wedge (b_{77} = 0) \wedge (b_{78} = 111) \wedge (b_{79} = 211) \wedge (b_{80} = 0) \quad (39)$$

The firing of t_B puts a token in p_B (see Figure 13(b)), whose colour is the vector defining the structure of the composed container, as previously discussed.

4 Conclusions and further research directions

The use of coloured Petri nets to represent logistics facilities and material handling systems in a multimodal hub compatible with the Physical Internet paradigm has been addressed in the paper. Petri nets are very suitable for modelling the activities that are carried out by the various π-resources when handling π-containers: from the simple tasks for moving goods within the hub to the complex processes that are actualized when a composed container has to leave the multimodal hub by road. Besides, the applicability of the Petri net formalism is not limited to the representation of a multimodal hub; they can be effectively adopted also to represent logistics networks with the aim of modelling and controlling the flows of π-containers through the various nodes of a network. In fact, one of the promised benefit to assess consists of a better exploitation of the transport capacity, as some mode might be filled with disaggregated π-containers but not with standard container. To do that with CPNs, further colour sets can be adopted in order to associate more detailed information (e.g., the destination of each π-container, the status of them, etc.) to the token representing the aggregations of π-containers travelling on the logistic networks.

In this connection, the current activities on this research topic are: to define and test the CPN model for the whole multimodal hub, and to propose a CPN model for a PI-compatible logistics network at interregional level. A further research direction is relevant to definition of specific simulation models, based on the CPN representation of both the multimodal hub and the logistics network, to carry out a structural and performance analysis and to optimize some parameters of the considered class of systems.

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