Resilience Assessment of Hyperconnected Parcel Logistic Networks Under Worst-Case Disruptions

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Abstract: *Logistics networks that shape the Physical Internet's logistics web are prone to potentially adversarial disruptions that impact their performance severely. In this article, we study a network interdiction problem on a multi-commodity flow network in which an adversary totally disrupts a set of network arcs with an intention to maximize the operational costs of delivering parcels. We formulate it as a two-stage mixed-integer linear program. To solve such complex large-scale program, we use linear programming duality and provide an algorithm based on the structure of the program that computes the set of network arcs that can be interdicted in order to reduce its size. Finally, we use the model and solution framework to evaluate the resilience capabilities of topology optimized hyperconnected networks and compare it with lean networks. We find that our developed solution methodology reduces the size of network interdiction program substantially and showcases superior computational performance against off-the-shelf optimization solvers. Furthermore, the resilience comparison between hyperconnected networks and lean networks depicts enhanced capabilities of the topology optimized hyperconnected networks to sustain worst-case disruptive events as opposed to that of lean logistics networks.*

Keywords: *Hyperconnected networks, Resilience, Worst-case disruptions, Stackelberg game, Physical Internet*

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Physical Internet Roadmap *(Link):☐ PI Nodes, ☒ PI Networks, ☐ System of Logistics Networks, ☐ Access and Adoption, ☐ Governance.*

1 Introduction

Due to increased internet accessibility to a wider population worldwide, there has been an upsurge in world-trade and e-commerce industry. This has ultimately led to a steep growth of the parcel delivery industry across the globe (Jin, 2018). This parcel delivery industry is inherently characterized by its reliance on numerous assets, such as hub facilities and vehicles, to deliver parcels to customers. These customers are spread out across a wide geographical region with an ever-increasing expectation of swift parcel delivery in a reliable manner. Such complex operations require meticulous planning and proper execution. Hub network design forms one of the major components of the planning phase (Cordeau et al., 2006). These hub facilities serve as parcel sortation and consolidation centers which are used to generate shipments and send them towards their respective destinations. Routing parcels through these hub facilities help consolidate them together thus yielding savings in transportation costs and greenhouse emissions and increasing the ability to provide better service between locations due to frequent transport connections.

Hub network design has been widely studied in the literature for the past thirty-five years (Campbell & O'Kelly, 2013) and hub-and-spoke networks have been recommended for parcel delivery industry. These hub-and-spoke networks are hierarchical networks which force parcel(s) to go through central hub(s) in order to consolidate them together and then deliver them to destination locations. This induces unnecessary travel for parcels and gives rise to parcel congestion at hub(s) at peak delivery times (Montreuil et al., 2018; Tu & Montreuil, 2019). In order to mitigate these issues, hyperconnected networks are proposed in the realm of Physical Internet (Montreuil, 2011). These hyperconnected networks are multi-plane open access interconnected hub meshed webs that link hubs at multiple planes. These densely connected meshed networks provide better degrees of freedom for parcel movement while retaining the benefits of cost savings achieved through parcel consolidation.

All hub networks, including hyperconnected networks, face various types of disruptions on a daily basis. These disruptions include but are not limited to traffic congestion on roads, power failure at hub facilities, hub throughput reduction due to pandemic, and total breakdown of network components in a region due to a natural disaster. These disruptive events lead to increased operational costs, late parcel deliveries, decreases in customer satisfaction, and ultimately lesser profit margins. Hence, it is essential to prepare against such disruptions. On a broader level, these disruptions can be classified as either total or partial disruptions (Mohammadi et al., 2016). A total disruption of a network component refers to the component being totally dysfunctional whereas a partial disruption of a component would render it functional but at a reduced capacity. All in all, due to the regular occurrence of such disruptive events across the hyperconnected networks, it is indeed paramount to develop a toolkit to assess the resilience of such networks.

There has been a growing interest among researchers regarding network resilience and its evaluations. For supply chain and logistics networks, one of the definitions of network resilience is the ability of a network to provide acceptable service level in presence of disruptions (Smith et al., 2011). One of the widely researched directions for resilience evaluation is through simulating disruptive events on the logistics networks to understand their performance under such events. Such experiments are usually termed as disruption experiments and they serve as a medium to understand the resilience properties of such network in depth (R. Li et al., 2017; Osei-Asamoah & Lownes, 2014). These experiments can simulate different types of disruptive events in which the network components fail either in a random manner or in a localized manner (Kulkarni et al., 2021). However, due to limitations in computing infrastructure, simulating all possible disruption events is pragmatically infeasible. To tackle such situations, analytical methods are developed to estimate the resilience of the pertinent network. These analytical methods study the network's structural properties, which estimate the vulnerability of the network and approximate its susceptibility to disruptions. These topological measures include but are not limited to node/edge degree, reachability, closeness centrality, betweenness centrality, short path lengths, and number of edge-disjoint paths (Bai et al., 2020; Ip & Wang, 2009, 2011; Kim et al., 2017; Kulkarni et al., 2021, 2022). Indeed, these simulation experiments or the topological measures provide a resilience assessment toolkit for stochastic disruptive events only.

Although useful, the above-mentioned disruption experiments require data about disruptive events in order to simulate them and evaluate resilience of the networks. However, availability of such data about the disruptive events is often limited which makes the results of the disruption experiments less accurate. One such instance is disruptions due to natural disasters.

As they occur very infrequently, having good quality data about them to prepare response actions is difficult. Nonetheless, as such disruptions cause significant harm to the networks, it is of paramount importance to be prepared for such events. Such disruptions are termed as worst-case disruptions and logistics networks are prone to them. As a result, there is a need to also develop a tool to assess resilience of the networks under such worst-case disruptive events. In such worst-case disruptions, a fictitious malicious attacker disrupts network components in such a way that causes maximum harm to the network operations. Such situations are modelled through game-theoretic approaches using optimization framework (Lim & Smith, 2007). In particular, such games known as Stackelberg games involve two-person decision making usually with conflicting objectives to attain and the problem is widely known as a network interdiction problem (Wood, 1993; Washburn & Wood, 1995). In this problem, the first player, a leader, is an adversary who fully or partially disrupts network components in order to block the follower's commodity flow. Such settings are modelled through two-stage optimization formulations and techniques from robust optimization are employed to solve them. Multiple variants of this problem are studied in the literature differentiated by the leader's objective. These include but are not limited to shortest path interdiction where leader aims to maximize the total shortest path length(s) that follower can use (Israeli & Wood, 2002); maximum flow interdiction where the leader aims to minimize the total commodity flow the follower can send in the network (Wollmer, 1964); minimum cost interdiction where the leader maximizes the cost that the follower face in order to fulfill the commodity demand in the network (Lim & Smith, 2007).

The contributions of the article are three-fold. First, we study the network interdiction problem through a Stackelberg (leader-follower) game where the leader (or interdictor) maximizes the parcel delivery costs and the followers objective is to minimize the costs after disruption. We formulate the said problem as a two-stage mixed-integer program where the outer problem belongs to the adversary where they maximize the cost of parcel delivery through interdicting transportation edges under a given interdiction budget. The inner decision problem belongs to the logistics company where they minimize the cost of parcel delivery operations, modelled as minimum-cost multi-commodity network flow problem after the edge interdictions. Our problem generalizes the shortest path interdiction problem proposed by (Israeli & Wood, 2002) to account for multiple commodities. Second, we analyze the structure of the problem and provide a two-phase methodology to solve the problem exactly. In the first phase we smartly deduce the network components that can be disrupted in the worst-case, which helps us reduce the size of the optimization formulation drastically. In the second phase we solve this new smaller optimization problem directly to obtain an optimal solution of the original problem. Third, we perform experiments to understand the computational performance of our solution approach, and then we compare the resilience capabilities of topology optimized hyperconnected networks generated through various methods (as proposed in the literature) with that of lean hyperconnected networks. The computational experiments depict the ability of developed solution methodology to efficiently evaluate the resilience of hyperconnected networks in worst-case disruptions as opposed to using optimization solvers. Next, these results indicate that the increase in operational costs during worst-case disruptions in topology optimized hyperconnected networks is lesser than that of lean hyperconnected networks. Moreover, the topology optimized hyperconnected networks are better able to maintain connectivity between the O-D pairs to allow parcel flow than the connectivity maintained by lean hyperconnected networks.

The rest of the paper is organized as follows: Section 2 describes the problem setting and formulates it through a two-stage mixed-integer linear program. Section 3 discusses the nature of the formulated problem and describes the solution methodology to solve the problem exactly. Next, we provide in Section 4 an in-depth computational study where we analyze the computational performance of the developed solution methodology against that of an optimization solver and compare the resilience of the hyperconnected networks with that of lean networks under adversarial disruptions. Finally, Section 5 lays out the concluding remarks and presents avenues of future research.

2 Problem Description

We consider a logistics company or group of such companies that delivers parcels across a given geographical region through its network. The company is interested in assessing the performance of its logistics network during a worst-case disruptive event, i.e., in a worst-case disruption scenario.

Formally, we consider a set of locations $\mathcal{S}$ where the parcel demand originates and a set of locations $\mathcal{T}$ where the parcels must be delivered. Let $\mathcal{P} \subseteq \mathcal{S} \times \mathcal{T}$ be the set of origin-destination (O-D) pairs of interest with each pair $p \in \mathcal{P}$ having a demand of d_p parcels. The company has opened a set of logistics hubs $\mathcal{H}$ at discrete locations to serve the O-D pairs. We consider the directed graph $\mathcal{G} = (\mathcal{S} \cup \mathcal{H} \cup \mathcal{T}, \mathcal{A})$ where commodities are transported from origins $\mathcal{S}$ through logistics hubs $\mathcal{H}$ to finally be delivered at destinations $\mathcal{T}$ through available transportations arcs $\mathcal{A} \subseteq (\mathcal{S} \cup \mathcal{H} \cup \mathcal{T})^2$. These logistics hubs serve as locations where the parcels are sorted and shipped towards their respective destinations. As these parcels are not stored for a longer duration at these hubs, the hub capacities are not restrictive. So, we assume that the hub has sufficient capacities to sustain the logistics operations and satisfy the demand. Moreover, due to the huge volume of parcel flows that the network faces, the parcel flow costs can be approximated through a linear flow function. Hence, for a parcel to traverse a transportation arc $(i, j) \in \mathcal{A}$, it faces a cost of $c_{i,j}$ per unit of parcel.

In order to evaluate the performance of the network under the worst-case disruption, we consider a fictitious adversary (leader) who intends to interdict network arcs to maximize the total parcel delivery cost. We assume that the adversary has a budget of totally interdicting β arcs in the network. We model this decision through a binary variable $x_{i,j} \in \{0,1\}$ that assumes a value 1 if the arc $(i, j) \in \mathcal{A}$ is interdicted or 0 otherwise. Next the logistics company (follower) responds to the disruption through routing the parcel flow in the network for each O-D accordingly to incur minimum possible cost. To this end, we define continuous parcel flow variables $f_{i,j}^p \geq 0$ for each transportation arc $(i, j) \in \mathcal{A}$ and between each O-D pair $p \in \mathcal{P}$. Now, the problem can be formulated as the following two-stage mixed-integer linear program:

$$E(\mathcal{P}, \mathcal{A}) = \max_x \min_f \sum_{p \in \mathcal{P}} \sum_{(i,j) \in \mathcal{A}} (c_{i,j} + M \cdot x_{i,j}) f_{i,j}^p \tag{1}$$

subject to:

$$\sum_{j \in \mathcal{T} \cup \mathcal{H} | (s,j) \in \mathcal{A}} f_{s,j}^p = d_p, \qquad \forall p = (s,t) \in \mathcal{P} \tag{2}$$

$$\sum_{i \in \mathcal{S} \cup \mathcal{H} | (i,t) \in \mathcal{A}} f_{i,t}^p = d_p, \qquad \forall p = (s,t) \in \mathcal{P} \tag{3}$$

$$\sum_{j \in \mathcal{T} \cup \mathcal{H} | (i,j) \in \mathcal{A}} f_{i,j}^p = \sum_{j \in \mathcal{S} \cup \mathcal{H} | (j,i) \in \mathcal{A}} f_{j,i}^p, \qquad \forall p = (s,t) \in \mathcal{P}, \forall i \in \mathcal{H} \tag{4}$$

$$\sum_{(i,j)\in\mathcal{A}} x_{i,j} \leq \beta \tag{5}$$

$$f_{i,j}^{p} \geq 0, \qquad \forall p=(s,t)\in\mathcal{P}, \forall(i,j)\in\mathcal{A}$$

$$x_{i,j} \in \{0,1\}, \qquad \forall(i,j)\in\mathcal{A}.$$

The outer problem represents the decision problem of the adversary who maximizes the total transportation cost of the parcels by interdicting network arcs. Constraint (5) restricts the number of interdicted arcs based on the available arc interdiction budget. The inner problem represents the response by the logistics company and is a multi-commodity minimum-cost-network-flow problem. It minimizes the total transportation costs of parcels after the arc interdictions. We allow parcels to flow through interdicted arcs but with a very high penalty of M units in addition to the regular transportation costs. It indeed incentivizes the parcels to travel along uninterdicted arcs unless no other alternative is present. Constraints $(2)-(4)$ enforce the commodity flow balance conditions at each node. Finally, the remaining constraints define the domain of the involved decision variables.

By solving this two-stage optimization problem, we can estimate the performance of logistics networks under worst-case disruptions. However, this program cannot be directly fed to optimization solvers due to its bilevel nature and is challenging to solve even for small sparse network instances. In the next section, we propose a solution methodology to optimally solve this problem for large-scale densely connected networks such as hyperconnected networks.

3 Solution Methodology

In order to develop a solution framework to solve the optimization problem described by equations $(1)-(5)$, we study the structure of the problem. A closer inspection of the problem indicates that the decision problem of the logistics company i.e., the inner decision problem is indeed a linear program due to the presence of continuous f variables only. As a consequence, we can leverage tools from linear programming duality theory to study the problem. Let π_i^p be the corresponding dual variables of constraints $(2)-(4)$, for every $p\in\mathcal{P}$ and every $i\in(\mathcal{S}\cup\mathcal{H}\cup\mathcal{T})$. Therefore, taking the dual gives us:

$$E(\mathcal{P},\mathcal{A}) = \max_{x,\pi} \sum_{p=(s,t)\in\mathcal{P}} d_p(\pi_s^p - \pi_t^p) \tag{6}$$

$$subject\ to:$$

$$\pi_i^p - \pi_j^p \leq c_{i,j} + M\cdot x_{i,j}, \qquad \forall p\in\mathcal{P}, \forall(i,j)\in\mathcal{A} \tag{7}$$

$$\sum_{(i,j)\in\mathcal{A}} x_{i,j} \leq \beta \tag{8}$$

$$x_{i,j}\in\{0,1\}, \qquad \forall(i,j)\in\mathcal{A}.$$

From this dualization procedure, the two-stage program is reduced to a lesser complex single-stage mixed-integer linear program. However, this newly obtained mixed-integer linear program $(6)-(8)$ comprises $(|\mathcal{P}|\times|\mathcal{A}|+1)$ constraints, which still are substantially high. One potential option to reduce the size of the problem through analyzing the adversary's behavior is via figuring out the set of network arcs $\mathcal{A}'\subseteq\mathcal{A}$ that can be interdicted in the worst-case disruption. To this end, we perform an analysis of the decision problem of the logistics company. As we know, it is a multi-commodity minimum-cost-network-flow problem. Due to absence of capacity restriction at logistics hubs, the minimum-cost route to deliver parcels for each O-D pair $p = (s, t) \in \mathcal{P}$ can be computed independently and will resemble the shortest path between the corresponding origin s and the destination t. Hence, given the interdicted arcs, the problem of the logistics company can be decomposed into $|\mathcal{P}|$ independent sub-problems with each sub-problem solved through finding the shortest O-D path on the available network after arc interdictions.

Now, let us understand which arcs will be interdicted in worst-case disruptions based on the interdiction budget. When $\beta = 0$, i.e., no arcs are interdicted, the logistics company will use the shortest path on the original graph $\mathcal{G}$ to transport parcels between each O-D pair. Let $\mathcal{A}_1$ be the set of such arcs. When $\beta = 1$, the adversary will interdict one of the network arcs from set $\mathcal{A}_1$ that will cause highest transportation cost increase. Now, when $\beta = 2$, the adversary will interdict at least one arc from $\mathcal{A}_1$ and the other (if budget allows) from the set of arcs $\mathcal{A}_2$ that comprises arcs of the second shortest paths between each O-D pair when one of the arcs from $\mathcal{A}_1$ is interdicted.

Indeed, this showcases that the network arcs that will be interdicted depends upon the available interdiction budget. It can be obtained by decomposing the problem for O-D pair p and using a search tree. The search tree iteratively obtains all permutations of arcs that can be interdicted and then subsequently computes the next shortest path for transportation of parcels. Algorithm 1 outlines the procedure to compute such arc set $\mathcal{A}'$. Once $\mathcal{A}'$ is obtained, we formulate this reduced problem $E(\mathcal{P}, \mathcal{A}')$ and feed directly to the optimization solver. In order to achieve even more speedups, we set $\pi_t^p = 0$ for every $(s, t) \in \mathcal{P}$ in $E(\mathcal{P}, \mathcal{A}')$ as π is invariant to translation.

Algorithm 1: Search strategy to find $\mathcal{A}'$

Input : Original Graph $\mathcal{G} = (\mathcal{S} \cup \mathcal{H} \cup \mathcal{T}, \mathcal{A})$, Interdiction budget β
Output : Subset of Arcs $\mathcal{A}' \subseteq \mathcal{A}$ that can be interdicted

Initialize: Set of arcs $\mathcal{A}' \leftarrow \emptyset$;
for every $(s, t) = p \in \mathcal{P}$ **do**
  Initialize: Set of search tree nodes $\mathcal{N} \leftarrow \emptyset$, Interdicted Arcs at each node $(\mathcal{I})_{n \in \mathcal{N}} \leftarrow \emptyset$, Tree depth at each node $(\mathcal{D})_{n \in \mathcal{N}} \leftarrow 0$, Children of each node $(\mathcal{C})_{n \in \mathcal{N}} \leftarrow \emptyset$, Shortest path at each node $(\mathcal{SP})_{n \in \mathcal{N}} \leftarrow \emptyset$, Copy of Graph at each node $(\mathcal{X})_{n \in \mathcal{N}} \leftarrow$ Copy of $\mathcal{G}$, nodes counter $m \leftarrow 0$;
  Append root node r: $\mathcal{N} \leftarrow \mathcal{N} \cup \{r\}$;
  while $\mathcal{N} \neq \emptyset$ **do**
    Select a node $n \in \mathcal{N}$ and remove the arcs $\mathcal{A}_n \leftarrow \mathcal{A}_n \setminus \mathcal{I}_n$ in the graph $\mathcal{X}_n$;
    $\mathcal{SP}_n \leftarrow$ Shortest path for between s and t in the graph $\mathcal{X}_n$;
    if $\mathcal{D}_n \leq \beta$ **then**
      for every $(i, j) \in \mathcal{SP}_n$ **do**
        $m \leftarrow m + 1, \mathcal{I}_m \leftarrow \mathcal{I}_n \cup \{(i, j)\}, \mathcal{C}_n \leftarrow \mathcal{C}_n \cup \{m\}, \mathcal{D}_m \leftarrow \mathcal{D}_n + 1, \mathcal{N} \leftarrow \mathcal{N} \cup \{m\}$, $\mathcal{A}' \leftarrow \mathcal{A}' \cup \{(i, j)\}$;
    $\mathcal{N} \leftarrow \mathcal{N} \setminus \{n\}$;
return $\mathcal{A}'$

4 Computational Study

In this section, we apply the developed resilience performance evaluation framework on various types of logistics networks. To this end, first we design multiple large-scale hyperconnected intercity parcel logistics hub networks to be the backbone infrastructure of China for ground transportation and consolidation of parcels. The networks are able to serve regions that house

93.58% of the Chinese population, are spread across 95.09% of the Chinese inhabitable land, and generate 94.42% of total Chinese GDP (Li et al., 2018). We design these networks through minimizing k-shortest paths (Kulkarni et al., 2021) and k-shortest edge-disjoint paths between each O-D pair (Kulkarni et al., 2022). We generate networks with number of paths, $k \in \{2,4\}$ and the hub cardinality, $N \in \{60,70,80,90\}$.

First, we show the value of the developed solution methodology in assessing the resilience performance of k-shortest edge-disjoint path networks. Table 1 depicts the computational performance of our methodology against that of the off-the-shelf optimization solver Gurobi when we assess the k-shortest edge-disjoint path networks of various cardinalities with an interdiction budget of 2. Due to the presence of large number of variables and constraints, Gurobi struggles to solve the problem and after 12 hours of execution as well is not able to obtain an optimal solution. In particular, the last retrieved optimality gaps in such cases are at least 500%, which showcases the best-known solution at the end of 12 hours is potentially far-off from the true optimal solution. On the other hand, we observe that the set of arcs that can be interdicted $\mathcal{A}'$ computed via Algorithm 1 is around half of the total arc set $\mathcal{A}$. As a consequence, this helps us reduce the size of the problem drastically. Specifically, the number of variables and constraints are reduced by at least 82% and 97% respectively. This reduced-size problem when next fed to Gurobi is solved in under a minute with total time required being under two minutes. Indeed, these results depict the efficacy of the developed solution methodology through which we can solve large-scale problems optimally and quickly. In turn, this methodology can be leveraged by any logistics company for resilience evaluation of their networks in the worst-case disruptions.

Table 1: Computational performance of developed solution methodology for k-shortest edge-disjoint path networks ($k = 2$) and interdiction budget $\beta = 2$.

Method	N	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{A}'\rvert$	# Variables	# Constraints	Total time (sec)	Optimality gap (%)
Algorithm 1 + Gurobi	60	1005	586	3137	4288	54	0
	70	1074	656	3227	4571	59	0
	80	1344	767	3312	4836	62	0
	90	1429	817	3403	5912	82	0
Gurobi	60	1005	-	18207	150,976	43200	504.5
	70	1074	-	18641	163,420	43200	831.4
	80	1344	-	21290	212,830	43200	1392.5
	90	1429	-	23388	261,777	43200	1965.2

Next, we compare the performance of these topology optimized hyperconnected networks with that of lean hyperconnected networks which are designed through efficiency considerations only. These lean networks are obtained through minimizing the (first) shortest path for each O-D pair with same hub cardinality as that of hyperconnected networks. Table 2 shows the comparison results of performance of these networks under various worst-case disruptions characterized by $\beta \in \{1,2,3,4,5\}$. We can observe that in all the networks, these worst-case disruptions fully disconnect multiple O-D pairs, which prevents the parcels from reaching the corresponding destinations. With an increase in the strength of the disruption event, we observe the number of O-D pairs that are disconnected increases. However, this effect is more nuanced. The topology optimized hyperconnected networks outperform the lean hyperconnected networks as they guarantee the flow of parcels for larger proportion of O-D pairs during a disruption event. In majority of cases, the disconnected O-D pairs for the k-shortest path or k-

shortest edge-disjoint path networks are half of their lean counterpart. This can be attributed to the presence of multiple paths that connect each O-D pair in the proposed hyperconnected networks. Moreover, the proposed hyperconnected networks guarantee this better O-D pair connectivity with overall lesser additional cost incurred during the worst-case disruptions for the connected O-D pairs. In particular, the percentage increase in costs for lean hyperconnected networks is at least two times more than that of topology optimized hyperconnected networks. Because these hyperconnected networks are designed through minimizing either multiple short paths or multiple edge-disjoint short paths, these hyperconnected networks have paths of comparable lengths which can be traversed during such disruption. The additional travel incurred in such cases is minimal, which is reflected through lower percentage increase in operational costs in hyperconnected networks. Importantly, in terms of both degradation metrics, the topology optimized hyperconnected networks worsen at comparatively slower rate than lean hyperconnected networks that worsen in a rapid manner. Overall, topology optimized hyperconnected networks outperform the lean hyperconnected networks substantially in terms of both maintaining better connectivity between the O-D pairs as well as lesser additional cost incurred under such adversarial disruptive event due to the presence of multiple short paths of comparable lengths.

Table 2: Performance of networks with size $N = 60$ in worst-case disruptions

Degradation Metric	Interdiction Budget (β)	Lean networks	k -shortest path networks		k -shortest edge disjoint path networks	
			$k = 2$	$k = 4$	$k = 2$	$k = 4$
# O-D pairs disconnected during the disruption	1	0	0	0	0	0
	2	3	3	0	1	0
	3	7	7	2	3	0
	4	16	13	11	10	6
	5	23	20	14	15	12
% Increase in costs for connected O-D pairs during the disruption	1	0.1096	0.06870	0.0027	0.19157	0.0265
	2	1.5051	0.48908	0.0584	1.63959	0.0763
	3	4.2091	2.06826	0.3765	1.03674	0.5112
	4	7.3841	2.20335	2.2176	0.00000	0.0000
	5	0.0000	2.33931	0.5587	0.00958	0.0042

Among k-shortest path networks or k-shortest edge-disjoint path networks, the latter in all disruptive events are able to maintain better connectivity between the O-D pairs; this is attributed to the presence of overall higher number of edge-disjoint paths that help maintain the connectivity. On the other hand, the additional costs incurred during the disruptions depict an interesting behavior. For lower disruption strength, specifically when $\beta \in \{1,2\}$, the k-shortest path networks depict lower additional costs increase whereas for higher strength of disruption, the k-shortest edge-disjoint path networks showcase better performance. As k-shortest path networks comprise of multiple paths of comparable lengths, it leads to lower increase in costs for lower strengths of disruptions. Moreover, because such paths have overlapping edges, and when such edges are disrupted in events with higher values of β, the additional cost incurred is considerably higher which doesn't occur in k-shortest edge-disjoint path networks.

Finally, we analyze the performance of hyperconnected designed with different values of k. Among these networks, we notice that with increase in value of k, the resilience capabilities of the networks also increase. As we decrease the value of k, higher number of O-D pairs are being disconnected and the increase is costs are higher as well. All in all, these worst-case disruption experiments corroborate the network performance predictions proposed by (Kulkarni et al., 2022) and demonstrate the value of considering network structure while designing hyperconnected networks to generate resilient logistics networks.

5 Conclusion

In this article, we motivate the need of assessing hyperconnected networks under adversarial attacks in the realm of Physical Internet. To the best of our knowledge, this paper serves as one of the initial investigations that provides a framework to assess the resilience of the hyperconnected networks in such worst-case disruptive events. We take inspiration from Stackelberg games and formulate a two-stage mixed-integer linear program that provides performance (in terms of costs) of a logistics network in the worst-case disruption. In order to solve the optimization problem optimally, we leverage tools from linear-programming duality theory to reduce the problem to a single-stage mixed-integer program. Furthermore, to reduce the size of the single-stage program, we study the arc(s) interdiction behavior and propose a search strategy that obtains the network arcs that can be interdicted. Finally, we solve the reduced single-stage program for the subset of arcs that can be interdicted through an off-the-shelf optimization solver to obtain the total operational costs faced by the network in the worst-case disruption.

In the computational study, we first design hyperconnected networks generated through either minimizing k-shortest paths or minimizing k-shortest edge-disjoint paths between each O-D pair as proposed in the literature. Next, we compare the computational performance of our developed solution methodology with that of off-the-shelf optimization solver through assessing the worst-case performance of these generated hyperconnected networks. We observe that off-the-shelf optimization is unable to obtain an optimal solution and terminates with huge optimality gaps due to the problem size. On the contrary, Algorithm 1 helps reduce the size of the original problem both in terms of number of variables and constraints substantially. In turn, it provides solution time speedups and is able to provide optimal solutions for even densely connected hyperconnected networks within few seconds. Indeed, this showcases the superior performance of the developed methodology which can be leveraged by logistics companies to evaluate the resilience of their networks under worst-case disruptive events.

Finally, we compare the worst-case performance of k-shortest path and k-shortest edge-disjoint path hyperconnected networks with that of lean hyperconnected networks. We find that topology optimized hyperconnected networks are able to maintain better connectivity between the O-D pairs and incur less additional operational costs during the worst-case disruption as compared to lean hyperconnected networks. Finally, we assess the resilience capabilities of hyperconnected networks and find with increase in value of the parameter k, the resilience of the networks increases. We notice that k-shortest edge-disjoint path networks are able to maintain better connectivity between the O-D pairs compared to that of k-shortest path networks. In terms of additional costs incurred for the connected O-D pairs, we observe that the k-shortest path networks showcase superior performance under worst-case disruption of lower intensity whereas with k-shortest edge-disjoint path networks depict better capabilities to sustain worst-case disruptions of higher degree.

The current work opens multiple avenues for future research. First, the hub capacity restrictions and parcel consolidation route generation can be embedded in the proposed modelling framework to capture the problem situation in a more realistic manner. Consequently, it motivates the need to develop tailored solution techniques that leverage the structure of the problem and concepts from the bilevel programming literature to solve such an optimization problem. Second, such worst-case disruptive events should be considered while designing hyperconnected networks in order to design networks with better resilience capabilities.

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