IPIC 2019 | 6th International Physical Internet Conference | London

User Equilibrium in a Transportation Space-Time Network

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Abstract: *We provide a method to obtain a User Equilibrium in a transportation network, in which we transport containers for multiple agents. The User Equilibrium solution is defined as the solution wherein each agent can travel via their cheapest paths possible, and no agent is harmed by the route choice of other agents. The underlying model used is the Space Time Network (STN), in which the travel time of modalities is fixed and independent of the occupancy of the network. The System Optimal solution is the solution in which the total costs of the network are minimised. An approach is presented to find a toll scheme to create a User Equilibrium solution in this tolled STN, while maintaining the System Optimal solution of the initial STN.*

Keywords: *User Equilibrium, System Optimal Solution, Space Time Network, Intermodal and Synchromodal Transport*

1 Introduction

In this paper we look at a transportation system where individual agents control logistic units. This occurs in Physical Internet and in Synchromodal networks. For an overview of those concepts and differences between them, we refer the reader to Ambra et al. (2019). The agents in such a network can be logistic service providers or clients controlling the stream of their containers, or intelligent containers or other smart logistic units themselves. Looking at the planning of such systems. We have to look at two aspects: information and the degree of control and optimisation. Both can take either a local view, where only own information is known and optimisation done is for an individual objective. Or a global view, where information is available for the entire network and the optimisation is aimed at a shared goal. We can distinguish (De Juncker et al., 2017) four different systems in a synchromodal framework, see Figure 1. If the information is available globally but every agent only optimises their own objective, we call the approach selfish. If the information is available globally and the decision is aimed to optimise the entire network, it is called social. If the information is only available locally and optimisation is also local, it is a limited approach. Lastly, if the decision is aimed at global optimisation with local information, we call it a cooperative approach.

In logistic (service) network planning problems Space-Time Networks (STN) are often used for the representation, see for example Andersen and Crainic (2009), Crainic (2000) and Del Vecchyo et al. (2018) . On this STN a non-negative integral Minimum Cost Multi-Commodity Flow problem (MCMCF) is solved to get the overall optimal (social) solution. However, if links have capacity constraints and there are multiple agents travelling or sending their commodities over the network, some agents may not receive the shortest or most economical path. They may be unhappy (in a selfish model) with the total solution, even when this solution is the optimal solution for all agents together, a system optimal solution. Note that all kind of modalities (or combinations) can be modelled using this approach.

To reach a solution in which all agents are satisfied, and do not want to change their paths, we would actually need a 'User Equilibrium' (UE) solution. However, in most cases this UE is overall a worse solution than the overall optimal 'System Optimal' (SO) solution. There is an expected gain (Roughgarden and Tardos, 2002) for the total system in case of cooperation, reaching a system optimal solution. Swamy (2007) shows that selfish, here meaning locally optimising, systems have their price: they prove that, in traffic assignment problems, travel times induced by selfish agents might be the same as the total travel time incurred by optimally routing twice as much traffic and indicate that adding central control or incentives gives an overall improvement of the system. However, in networks with high load the performance might not suffer too much, as can be found in Peeta and Mahmassani (1995). So optimising the total network and then sharing the benefits from an overall optimal solution between all agents is beneficial for all. On the other hand, it is not easy, as it requires a mental shift to get to give up control.

Fig. 1: Different models of a synchromodal network.

In this work we propose for the first time a definition of a *UE solution in a logistic STN*. We then *provide a method how to change the arc weights of the STN to create and find a UE solution in the modified STN*, by adding tolls, *that equals the system optimal solution*. Note that the practical implementation is far away, but this can be used to propose a reallocation of costs in which the benefit of the social optimal, with respect to the UE in the original STN, is shared in a fair way. In terms of Figure 1, we want to get the 'social' solution in a 'selfish' network.

For the second part, changing the arc weights to create a UE solution that equals the SO solution, we propose the following algorithm. The first step in this 'all toll algorithm' is to calculate the SO based on the path costs of agents travelling from their origin to their destination. The next step is to calculate tolls that are added to the paths in the network. These tolls are used to adjust the path costs, such that we can offer the agents a choice of tolled paths.

Now, when the agent gets assigned its cheapest tolled paths, those paths are in the SO solution and the solution is UE as well. The solution is a UE because the offered path costs are the cheapest option according to the information available for the agent, the new tolled STN.

In the next section we discuss the literature on User Equilibria and toll systems in traffic assignment problems. To the best of the authors' knowledge no literature exists for UE in freight logistic networks. In Section 3 the definition of UE in STN is given and a method is presented to find a UE that equals the solution of the system optimal. The method is illustrated by two examples in Section 4. We conclude with some remarks and directions for future research.

2 Literature review

Most of the literature about User Equilibria is based on network congestion, where travel times on roads depend on occupancy of travelling arcs, as in traffic assignment problems. Van Essen et al. (2016) give a proper review of ways to force a UE into a System Optimum (SO) by diffusing travel information to stimulating some agents to travel non-selfishly to achieve cheaper total costs. Peeta and Mahmassani (1995) investigate both the SO and the UE Time-Dependent Traffic Assignment. They show that the more goods have to be transported, the more the solutions of the two models differ from each other. Bar-Gera (1999) provides a solution method for the UE traffic assignment problem which is computationally efficient, memory conserving and an origin-based solution method. Xu et al. (2012) propose a stochastic UE for a passenger transport network.

Miyagi et al. (2012) consider a traffic assignment problem from the view of game theory. They assume drivers have knowledge of the network and a Nash Equilibrium (which corresponds to a UE) is achievable. Wagner (2014) shows that the existence of a Nash Equilibrium is guaranteed under some natural assumptions on the travel time models. Also Wang and Yang (2017) show the equality of Nash Equilibrium and UE. Levy et al. (2016) consider selfish agents in a traffic assignment problem, and apply properties of game theory on traffic problems. They start from finding a UE solution, in which all agents take the best route for themselves, based on their route choice experiences in the past. The question then is if it is possible to obtain a System Optimal solution, in which agents are still selfish.

The relationship between the UE and the System Optimal can be examined by the Price of Anarchy (Roughgarden, 2006), a system often used in both economics and game theory, that measures how the efficiency of a system degrades due to selfish behaviour of its customers. Bar-Gera et al. (2012) consider the UE problem with the focus on spreading flow over the network (not time-dependent). They also introduce several criteria which can be taken into consideration for choosing UE solution methods. Their most important addition to the subject is the condition of proportionality: the same proportions apply to all travellers facing a choice between a pair of alternative paths, regardless of their origins and destinations.

Corman et al. (2015) consider the application of multimodal transport to provide a UE solution, with the choice of modality based on the wishes of the agents. They assume that agents have access to a system for publishing demand and offering transportation possibilities. Moreover, they assume everybody has access to truck transportation, so transport is always possible, regardless of the fact that other modalities are not available. They define every agent as one unit of transport, which has to choose one specific mode for the whole travel distance. The goal is to assign agents to modes in such a way that no agent will change its departure time and its route (and thus will not change its mode), to provide all agents a sufficiently good route.

One commonly used approach for creating a UE is by using tolls. Hearn and Ramana (1998) make use of a toll pricing system by adding a toll term to the cost function for each arc. They also describe the Robinhood formulation, in which the sum of all tolls must be zero, so that there is no profit for the system. In this case they calculate the toll after a System Optimal solution is found. According to Florian and Hearn (2003), the application of those types of toll is hard to implement on traffic networks regarding variable travel times, although the selective use of negative tolls to influence route choice of users might have some appeal. Yang and Han (2008) investigate the use of tolls with the help of the price of anarchy. Yang and Zhang (2008) constructed an anonymous link toll system to add traveller-dependent tolls. They concluded that there exist nonnegative links tolls identical to all users to decentralise the Wardropian System Optimum as a UE-CN (Cournot-Nash) mixed equilibrium, and the valid toll set is made up of a convex set of linear equalities and inequalities. They use nonnegative tolls only. Yang and Huang (2005a) state that Value Of Time (VOT) is a very important concept in transportation system modelling. The VOT of an order is a constant which denotes the importance of that agent. Didi-Biha et al. (2006) also use nonnegative tolls. Their goal is to maximise the toll revenue for the highway authority while the users of the network want to minimise their travelling costs. They introduce their bi-level programming Toll Optimisation Problem, both arc, arc-path and path based.

Yang and Huang (2005b) proved the existence of a Pareto refunding scheme that returns the congestion pricing revenues to all users to make everyone better off. This Pareto refunding scheme refunds class-specific and OD-specific toll revenue equally to all users in the same Origin-Destination pair in the same user class.

User Equilibria in (multi-) agent environments are also described as consensus seeking agents. Work on this is done by Ren and Beard (2005) and Liu and Liu (2012).

3 User equilibrium in STN

In this paper we propose the use of tolls on paths within as STN. For convenience we will use the terms agent for the controller of (at least) one unit of transport. This agent can send an order (multiple units) for transportation within the network and as unit we will say container. A Physical Internet system or an other self-organising system with smart units will fit within this method.

For each order there may be multiple paths to travel by, within the STN. We propose the following definition for a UE within an STN:

Definition 1 (User Equilibrium) A UE in an STN is reached when each agent can use their cheapest paths.

This is obviously not always possible when concerning only the initial networks, so we need to adjust the initial network using the path tolls to reach this UE. We will assume that agents are not familiar with the path costs in the initial STN, they only have knowledge of the tolled path costs. In this section, our goal is to find a Path Tolled UE. Our approach is to first find an SO solution, and then add tolls to paths, which create a new cost scheme for paths. When we offer the STN with the adjusted path costs to the agents, they can selfishly choose routes, and the outcome of their path choice will correspond with the path to order assignment within the SO. The difference between the two networks provides insight in the offered fairness by the solution and can be used to redistribute the gain between the agents.

The way of finding tolls which give us a UE solution in an initial SO problem is described in Algorithm 1, which is partly based on the solution algorithms used by Hearn and Ramana (1998) and Jiang and Mahmassani (2013). The difference with the framework of Hearn and Ramana is that we do not define the toll set, because in our approach there is no need to obtain this total set. The difference with Jiang and Mahmassani (2013) is that we apply tolls on paths instead of updating path assignment.

The Space-Time Network is a directed graph $G = (V,A)$, consisting of a set of nodes $v \in V$ and a set of directed arcs $a \in A$. Each arc a is a link between two nodes, an origin node v_1 and an end node v_2: $a = (v_1,v_2)$, along which a container can travel. We use x_a to denote the number of units of flow along arc a. An Origin-Destination-pair (OD-pair) w is a pair of two nodes, origin location w_O and destination location w_D, so $w = (w_O,w_D)$, which is not necessarily an arc. The number of containers an order wants to transport from w_O to w_D is denoted by d_w, the demand of order w. A path p consists of a sequence of (non-horizontal) adjacent arcs between two nodes. In our problem we only consider paths between origin and destination nodes. $\underline{f_p}$ denotes the path flow of path p (always integer), with $p \in P_w$, $w \in W$, where $\mathcal{P}_w$ is the set of all paths for OD-pair w and W is the set of all OD-pairs. The total path set is $\mathcal{P} \coloneqq \bigcup_{w \in W} \mathcal{P}_w$. The costs of an arc a are denoted by c_a and the path costs of path p are denoted by C_w^p or C^p. The capacity of an arc is denoted by m_a and the capacity of a path is denoted by m_p. The available arcs in a path are denoted by

$$\delta_{ap} = \begin{cases} 1 \text{ if } a \text{ is contained in } p, \forall a \in A, p \in \mathcal{P} \\ \quad 0 \text{ otherwise} \end{cases} \tag{1}$$

After finding the SO solution, we want to find path tolls (β_w^p) such that each agent is satisfied with its route, and thus a UE is achieved. We only use tolls to obtain both an SO and UE solution, so we do not need to make profit on the tolls. We will search for tolls that are as low as possible and we require that all tolls payed or received by agents sum up to zero. We will now go through the proposed algorithm. Finding the path tolls starts with an SO solution (Step 1), solving of which results in the optimal flows $\underline{f_p}$ (Step 2). Now, define the set of paths used in the SO solution (Step 3) by $h_{in,w} \coloneqq \left\{p \,\middle|\, \underline{f_p} > 0,\ p \in \mathcal{P}_w\right\}$, and the sets of all other paths (which are not in the SO solution) by $\mathrm{h}_{\mathrm{out,w}} \coloneqq \left\{p \,\middle|\, \underline{f_p} = 0,\ p \in \mathcal{P}_w\right\}$.

We then solve a Nonlinear Programming Problem NP-β that consists of an objective function that minimises the path tolls of a certain path set, and a set of constraints. To realise low tolls on paths in $h_{out,w}$ we will minimise the tolls added to paths which are not in the SO solution, so we use as the objective function:

$$\sum_{\mathrm{w} \in \mathcal{W}} \sum_{p \in h_{out,w}} \left|\beta_w^p\right|. \tag{2}$$

To let the tolls sum up to zero we use the constraint:

$$\sum_{p \in h_{in,w}} \beta_w^p\, \underline{f_p} = 0. \tag{3}$$

So, if there are tolls needed to obtain a UE, there will be one or multiple agents who need to pay toll, as well as there are one or multiple agents who receive toll. This last group thus has a discount on the routes which we want those agents to take. We do not want the toll received by an agent to be higher than the initial path cost (which would mean that an agent does not have to pay, but only receives money for choosing a certain path), so we use the constraints:

$$C_w^p + \beta_w^p \geq 0 \; \forall\, p \in \mathcal{P} \quad \Leftrightarrow \quad \beta_w^p \geq -C_w^p \; \forall\, p \in \mathcal{P}. \tag{4}$$

Now, the NP-β (step 4) consists of the following constraints:

$$\sum_{w \in \mathcal{W}} \sum_{p \in h_{in,w}} \beta_w^p \underline{f_p} = 0 \tag{5}$$

$$\beta_w^i - \beta_w^j \leq C_w^j - C_w^i \; \forall\, (i,j),\; i \in h_{in,w},\; j \in h_{out,w} \quad \forall\, w \in \mathcal{W} \tag{6}$$

$$\beta_w^p \geq -C_w^p \quad \forall\, p \in \mathcal{P} \tag{7}$$

where Constraint (5) ensures that all tolls on paths used in the SO solution sum up to zero, Constraint (6) ensures the paths used in the SO solution for one order, have equal or lower costs than the paths for that order which are not in the SO solution, and Constraint (7) ensures no tolled cost can become negative.

The NP-β (Step 4) is non-linear, which makes this problem hard to solve. We therefore use the equivalent linear formulation of the problem:

$$\min \sum_{w \in \mathcal{W}} \sum_{p \in h_{out,w}} \gamma_w^p \tag{8}$$

$$\text{s.t.} \quad \sum_{w \in \mathcal{W}} \sum_{p \in h_{in,w}} \beta_w^p \underline{f_p} = 0 \tag{9}$$

$$\beta_w^i - \beta_w^j \leq C_w^j - C_w^i \; \forall\, (i,j),\; i \in h_{in,w},\; j \in h_{out,w} \quad \forall\, w \in \mathcal{W}, \tag{10}$$

$$\beta_w^p \geq -C_w^p \quad \forall\, p \in \mathcal{P}, \tag{11}$$

$$\beta_w^p \leq \gamma_w^p \quad \forall\, p \in \mathcal{P}, \tag{12}$$

$$-\beta_w^p \leq \gamma_w^p \quad \forall\, p \in \mathcal{P}, \tag{13}$$

$$\gamma_w^p \geq 0 \quad \forall\, p \in \mathcal{P}, \tag{14}$$

where γ_w^p replaces the absolute value variable $|\beta_w^p|$.

Solving the NP-β (step 5) leads to toll that can be used to change the path costs (Step 6). The desired outcome of Algorithm 1 is that the solution to the SO-β problem is equal to the initial SO problem (Step 7). The resulting path costs are the only costs that are showed to the agents, so the agents do not have any knowledge about the initial STN and those path costs.

Algorithm 1 Calculating path tolls

1: Create SO problem:

$$\min \sum_{p \in \mathcal{P}} C_w^p f_p \tag{15}$$

$$s.t. \; x_a = \sum_{p \in \mathcal{P}} \delta_{ap} f_p \quad \forall\, a \in \mathcal{A}, \tag{16}$$

$$\sum_{p \in \mathcal{P}_w} f_p = d_w \quad \forall\, \mathrm{w} \in \mathcal{W}, \tag{17}$$

$$x_a \leq m_a \quad \forall\, a \in \mathcal{A}, \tag{18}$$

$$f_p \in N_0 \quad \forall\, p \in \mathcal{P}, \tag{19}$$

$$x_a \in N_0 \quad \forall\, a \in \mathcal{A}. \tag{20}$$

2: Solve SO problem, output: path flow vector *f*.
3: Create two lists for each order *w:*

$$h_{in,w} = \left\{p \,\middle|\, \underline{f_p} > 0,\ p \in \mathcal{P}_w\right\},\ h_{out,w} = \left\{p \,\middle|\, \underline{f_p} = 0,\ p \in \mathcal{P}_w\right\}.$$

4: Create NP-β:

$$\min \sum_{w\in W} \sum_{p\in h_{out,w}} \gamma_w^p \quad (21)$$
$$\text{s.t.} \sum_{w\in W} \sum_{p\in h_{in,w}} \beta_w^p \underline{f_p} = 0 \quad (22)$$
$$\beta_w^i - \beta_w^j \le C_w^j - C_w^i\ \forall (i,j),\ i \in h_{in,w},\ j \in h_{out,w} \qquad \forall w \in \mathcal{W}, \quad (23)$$
$$\beta_w^p \ge -C_w^p \qquad \forall p \in \mathcal{P}, \quad (24)$$
$$\beta_w^p \le \gamma_w^p \qquad \forall p \in \mathcal{P}, \quad (25)$$
$$-\beta_w^p \le \gamma_w^p \qquad \forall p \in \mathcal{P}. \quad (26)$$

5: Solve NP-β, output: β_w^p.

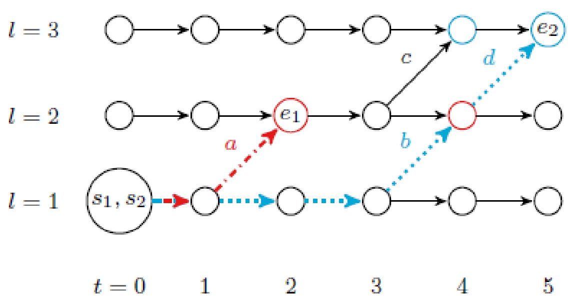
6: Add tolls β_w^p to the SO problem, SO-β:

$$\min \sum_{p\in\mathcal{P}} (C_w^p + \beta_w^p) f_p \quad (27)$$
$$\text{s.t. } x_a = \sum_{p\in\mathcal{P}} \delta_{ap} f_p \qquad \forall a \in \mathcal{A}, \quad (28)$$
$$\sum_{p\in\mathcal{P}_w} f_p = d_w \qquad \forall w \in \mathcal{W}, \quad (29)$$
$$x_a \le m_a \qquad \forall a \in \mathcal{A}, \quad (30)$$
$$f_p \in N_0 \qquad \forall p \in \mathcal{P}, \quad (31)$$
$$x_a \in N_0 \qquad \forall a \in \mathcal{A}. \quad (32)$$

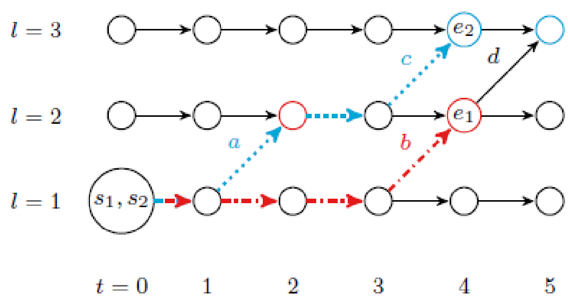
7: Solve SO-β problem, output path flow vector f.

4 Numerical examples

We illustrate the algorithm, by solving two examples. In the first example there are three locations, V = {1,2,3}, and five time steps. We have two connections between location $l = 1$ and $l = 2$ and two between $l = 2$ and $l = 3$. Those arcs all have capacity $m_a = 1$, and $m_a = \infty$ for (horizontal) waiting arcs. We have two orders, order 1 and 2 both start at location 1, order 1 has to go to $l = 2$ and order 2 to $l = 3$. Every node column shows a time step and each arc has cost $c_a = 1$. The two possible solutions are given in Figure 2, with s_w denoting the starting point and e_w denoting the end point for order w.



(a) The SO solution.



(b) The alternative solution.

Fig. 2: STN with two orders, first example..

In Figure 2a we see the SO solution, resulting from Step 1 and Step 2, that is the solution where the total costs are minimized. Here order 1 is delivered first with cost $C_1^a = 2$ and therefore order 2 can only take path *bd* with cost $C_2^{bd} = 5$. In Figure 2b a solution is given where both orders pay cost 4, that is path *b* for order 1, and path *ac* for order 2.
We can see that each order has its own preferable solution, that is the one in which they can travel via arc a, which is in the cheapest path for both orders. We have path costs $C_1^a = 2$, $C_1^b = 4$, $C_2^{ac} = 4$, $C_2^{ad} = 5$ and $C_2^{bd} = 5$, and the path sets following from the SO solution as obtained in Algorithm 1 in Step 3: $h_{in,1} = \{a\}$, $h_{in,2} = \{bd\}$, $h_{out,1} = \{b\}$, $h_{out,2} = \{ac, ad\}$.

The tolls given by Step 4 and Step 5 are $\beta_1^a = 1$, $\beta_2^{bd} = -1$, so all tolls on paths $p \in \bigcup_{w \in \mathcal{W}} h_{out,w}$ are zero and so is the objective value of the NP-β. The best solution of the NP-β is indeed the solution as obtained from Algorithm 1 Step 5: $\beta_1^a = 1$, $\beta_1^b = 0$, $\beta_2^{ac} = 0$, $\beta_2^{ad} = 0$, $\beta_2^{bd} = -1$ and with those tolls we obtain the path costs: $C_{\beta 1}^a = 3$, $C_{\beta 1}^b = 4$, $C_{\beta 2}^{ac} = 4$, $C_{\beta 2}^{ad} = 5$ *and* $C_{\beta 2}^{bd} = 4$, Both orders can travel via their cheapest paths, so both an SO and a UE are obtained.

In the second example (Figure 3) we have three orders, all with different demand: $d_1 = 3$ from location 1 to 2, $d_2 = 3$ from location 1 to 3 and $d_3 = 1$ from location 2 to 3. An SO solution is given in Figure 2, with s_w and e_w denoting the start end point of order *w*, respectively. All traveling arcs have capacity 1, except for arcs *a*, *c* and *f*, which have capacity $m_a = m_c = m_f = 2$, what we graphically show by multiple arcs between a pair of nodes.

We have path costs

$$C_1^a = 1,\ C_1^b = 2,\ C_1^c = 3,\ C_1^d = 5,$$

$$C_2^{ae} = 2,\ C_2^{af} = 3,\ C_2^{ag} = 4,\ C_2^{ah} = 5,\ C_2^{bf} = 3,\ C_2^{bg} = 4,\ C_2^{bh} = 5,\ C_2^{cg} = 4,\ C_2^{ch} = 5,$$

$$C_3^e = 2,\ C_3^f = 3,\ C_3^g = 4,\ C_3^h = 5,$$

The path sets following from the SO solution are:

$$h_{in,1} = \begin{Bmatrix} a, & c, & d \\ 1, & 1, & 1 \end{Bmatrix}, \qquad h_{out,1} = \begin{Bmatrix} a, & b, & c \\ 1, & 1, & 1 \end{Bmatrix},$$

$$h_{in,2} = \begin{Bmatrix} ae, & bf, & cg \\ 1, & 1, & 1 \end{Bmatrix}, \qquad h_{out,2} = \begin{Bmatrix} af, & ag, & ah, & bf, & bg, & ch \\ 2, & 1, & 1, & 1, & 1, & 1 \end{Bmatrix}$$

$$h_{in,3} = \begin{Bmatrix} f \\ 1 \end{Bmatrix}, \qquad h_{out,3} = \begin{Bmatrix} e, & f, & g, & h \\ 1, & 1, & 1, & 1 \end{Bmatrix}.$$

,

We see that none of the orders can travel via their cheapest paths, so we need tolls to create a UE. Solving the NP-β gives us $\beta_1^a = 2\frac{1}{3}$, $\beta_1^c = \frac{1}{3}$, $\beta_1^d = -1\frac{2}{3}$, $\beta_2^{cg} = -1$, $\beta_2^{ae} = 1$, $\beta_3^f = -1$, $\beta_1^b = 1\frac{1}{3}$. Note that path $b \in h_{out,1}$, so the toll on that path is not actually payed.

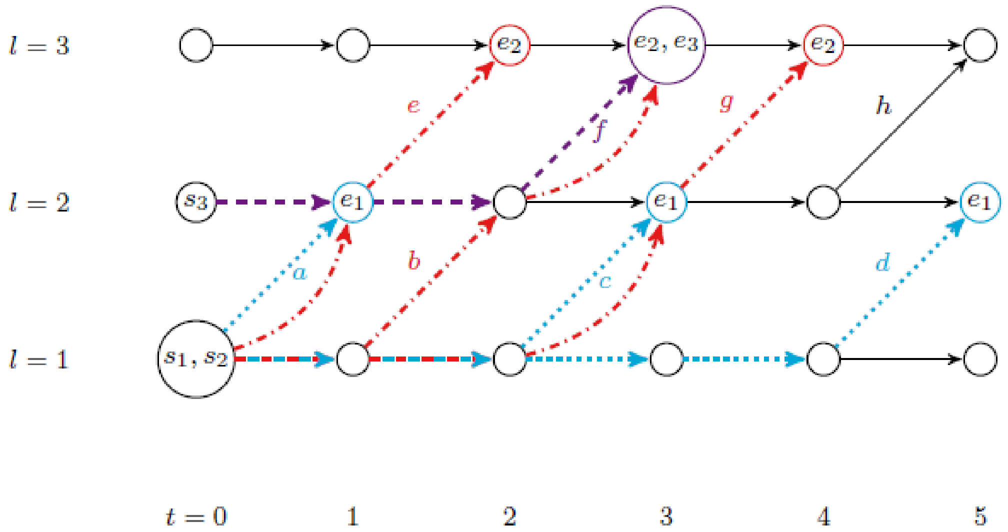


Fig. 3: STN with two orders, second example.

Table 1: Values $h_{out,w}$.

Order	1			2						3			
$p \in h_{out,w}$	*a*	*b*	*c*	*af*	*ag*	*ah*	*bf*	*bg*	*ch*	*e*	*f*	*g*	*h*
Initial path costs	1	2	3	3	4	5	3	4	5	2	3	4	5
Tolls	2⅓	1⅓	⅓	0	0	0	0	0	0	0	−1	0	0
Resulting path costs	3⅓	3⅓	3⅓	3	4	5	3	4	5	2	2	4	5

Table 2: Values $h_{in,w}$.

Order	1			2			3
$p \in h_{in,w}$	*a*	*c*	*d*	*ae*	*bf*	*cg*	*f*
Initial path costs	1	3	5	2	3	4	3
Tolls	2⅓	⅓	-1⅔	1	0	−1	−1
Resulting path costs	3⅓	3⅓	3⅓	3	3	3	2

5 Conclusions

The goal of this work was to provide a method to obtain a User Equilibrium in a logistic, intermodal or synchromodal Space Time Network (STN), in which we transport containers for multiple agents. We defined a UE as the solution where each agent can send its containers via its cheapest paths. We expanded this goal to also finding a solution of assigning containers to modes where the solution is System Optimal and by adding tolls a UE simultaneously. The first step in all toll algorithms is to calculate the SO based on the path costs of containers travelling from their origin to their destination. The next step is to calculate tolls that are added to the path or order costs, depending on what kind of tolls we considered.

When applying path based tolls, we assume agents do not know the path costs of the initial network (and thus also do not know their initial cheapest paths). Here the tolls are used to adjust the path costs, such that we can offer the agents a choice of tolled paths. Then when the agent gets assigned its cheapest tolled paths, those paths are in the SO solution and the solution is UE as well. The solution is UE because the offered path costs are the cheapest option according to the information available for the agent. We succeeded in finding an

approach to obtain both an SO and a UE solution on an STN. An practical note here is that using this approach in a real system with selfish agents is not easy. However, we think that this method can be used for sharing the benefits coming from a centrally controlled network.
For further research, we propose to take due dates into account. When we do this, it can be the case that orders will arrive too late compared to this due date. We then need to add a penalty function to the cost objective function in order to minimise the number of orders arriving too late. With the tolls, it is possible to share the penalty costs by all orders who are causing the lateness of the delayed orders. Another aspect that should be looked at is fairness of the UE solution. In the presented approach a UE is found and the benefit of the SO is shared between the agents. We do not know, however, whether this sharing is done in the fairest way.

Acknowledgements

This work has been carried out within the project ‘Complexity Methods for Predictive Synchromodality’ (Comet-PS), supported and funded by NWO (the Netherlands Organisation for Scientific Research), TKI-Dinalog (Top Consortium Knowledge and Innovation) and the Early Research Program ‘Grip on Complexity’ of TNO (The Netherlands Organisation for Applied Scientific Research).

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